Math 8
Worksheet 4

Conditionals

The last major pair of logical connectors is the conditional $P \implies Q$ and the biconditional $P \iff Q$ (our book uses single arrows $P \to Q$ and $P \leftrightarrow Q$ for these; this is just a difference in notation).

The first is read "if P, then Q", "P is a sufficient condition for Q", or "Q is a necessary condition for P", and is true when P is false (so the condition is not satisfied) or P and Q are both true (so the condition Q is true whenever the condition P is). This is logically equivalent to $\neg P \vee Q$ and $\neg Q \implies \neg P$.

The second is read "P if and only if Q" and sometimes written "P iff Q", and is true when P and Q have exactly the same truth value (both true or both false). It is logically equivalent to $(P \implies Q) \wedge (Q \implies P)$, $(P \implies Q) \wedge (\neg P \implies \neg Q)$, $(P \wedge Q) \vee (\neg P \wedge \neg Q)$, and $\neg(P + Q)$.

The *converse* of an implication $P \implies Q$ is $\neg P \implies \neg Q$, and the *contrapositive* is $\neg Q \implies \neg P$. A conditional is equivalent to its contrapositive, but not to its converse; a biconditional is equivalent to a conditional and its converse both being true.

1: Simplify the following expressions:

(a) $(P \iff Q) \wedge \neg Q$

(b) $(P \iff Q) \wedge (P + Q)$

(c) $(P \implies Q) \vee (P \implies R)$

(d) $(P \wedge Q) \implies (P \iff Q)$

(e) $((P \implies Q) \implies R) \wedge (P \implies (Q \implies R))$

2: Analyze the logical form of the following statements. If they are arguments, are they valid? (Problems (a) through (d) are exercise 1 on page 53 of Velleman.)

(a) If this gas either has an unpleasant smell or is not explosive, then it isn't hydrogen.

(b) Having both a fever and a headache is a sufficient condition for George to go to the doctor.

(c) Both having a fever and having a headache are sufficient conditions for George to go to the doctor.

(d) If $x \neq 2$, then a necessary condition for x to be prime is that x be odd.

(e) An integer n is either even or odd (but not both). If n is odd, then $n+1$ is even. If n is even or m is even, then nm is even. Thus, $n(n+1)$ is even.

(f) A sufficient condition for a function f to be continuous is for f to be constant. A necessary condition for f being differentiable is to be continuous. Thus, a discontinuous function f is not constant or not differentiable.

(g) A square s is in either set A or set B (but not both). If $s \in A$, then the triangle t is in set B. Thus, $A \neq \emptyset$.