Math 8
Worksheet 6

More Set Theory

Set elements need not be "simple" objects like numbers. Sets of sets are often useful to consider. The most important is the *power set*: the power set of a set A is $\mathscr{P}(A) = \{S : S \subseteq A\}$. That is, $(\forall S \subset U)(S \in \mathscr{P}(A) \iff S \subseteq A)$.

We also talk about *indexed families* of sets: if for every element i of some *index set* I we have a set A_i, the indexed family is $\mathscr{A} = \{A_i : i \in I\}$. The family intersection $\bigcap \mathscr{A} = \bigcap_{i \in I} A_i = \{a : (\forall i \in I)(a \in A_i)\}$ and family union $\bigcup \mathscr{A} = \bigcup_{i \in I} A_i = \{a : (\exists i \in I)(a \in A_i)\}$ generalize the operations we have defined for pairs of sets.

1: Let $I = \{1, 2, 3\}$ and $A_i = \{x \in \mathbb{Z} : i \leq x \leq 2i\}$.

(a) What is $\mathscr{P}(A_i)$ for each i?

(b) Why does $\mathscr{P}(\bigcap_{i \in I} A_i) = \bigcap_{i \in I}(\mathscr{P}(A_i))$? What is it?

(c) Why doesn't $\mathscr{P}(\bigcup_{i \in I} A_i) = \bigcup_{i \in I}(\mathscr{P}(A_i))$? [Give an example of an element in one and not the other, and argue why that is the case.]

2: Working in the universe $\mathbb{N}$, for all $n \in I = \{m : m \geq 2\}$ define the sets $M_n = \{m : m > n\} \cap \{p : (\exists q)(p = nq)\}$.

(a) Describe the members of the set M_n (in words).

(b) Prove that $\forall n \forall m (M_n \cap M_m \neq \emptyset)$.

(c) Prove that $\bigcap_{n \in I} M_n = \emptyset$.

(d) Prove that $\bigcup_{n \in I} M_n \neq \mathbb{N}$.

(e) What is the set $\mathbb{N} \setminus \bigcup_{n \in I} M_n$ (in words)? [Try writing out the first few terms.]