Math 8
Worksheet 10

Proofs IV

1: Prove the following:

(a) (Velleman pg. 135 n. 21) Suppose $\mathcal{F}$ and $\mathcal{G}$ are families of sets. If $\bigcup\mathcal{F} \nsubseteq \bigcup\mathcal{G}$, then there is some $A \in \mathcal{F}$ such that for all $B \in \mathcal{G}$, $A \nsubseteq B$.

(b) (Velleman pg. 134 n. 12) For all $x \in \mathbb{R}$, $x \neq 1$ is equivalent to there being $y \in \mathbb{R}$ such that $x + y = xy$.

(c) Suppose $|x - 1| < 1$. Then $x > 0$.

(d) If $3 \mid n$, then the remainder of $n \div 6$ is either 0 or 3.

2: Give a counterexample to the following “theorems”.

(a) If a, b, and c are real numbers, the polynomial $ax^2 + 2bx + c$ has two distinct real roots.

(b) Suppose $x \neq 0$ is given. Then for any real number y, there exists a real number z such that $z^2 < x + y$.

(c) If $\bigcup \mathcal{F} \subseteq \bigcup \mathcal{G}$, then there exists some $A \in \mathcal{F}$ and $B \in \mathcal{G}$ such that either $A \subseteq B$ or $B \subseteq A$.