Math 8
Worksheet 11

Proofs V

1: Prove the following:

(a) If n^2 is odd, then $n^2 - 1$ is divisible by 8.

(b) If p and q are prime, $p \mid n$, and $q \mid n$, then $pq \mid n$.
[You may use without proof the fact that if $p \mid ab$ and p is prime, then $p \mid a$ or $p \mid b$. Is this true if p and q can be any integers?]

(c) For any real number x, $|x| \leq 1 + x^2$. [Consider cases based on $|x|$.]

(d) For any real number x, $|x| \leq \frac{1}{2}\left(1 + x^2\right)$. [As scratch work, consider manipulating the desired inequality to recognize a non-negative quantity.]

2: A function $f : \mathbb{R} \to \mathbb{R}$ is *continuous* if

$$(\forall x)(\forall \epsilon > 0)(\exists \delta > 0)\Big(\big|y - x\big| < \delta \implies \big|f(y) - f(x)\big| < \epsilon\Big).$$

Prove that if $g : \mathbb{R} \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are both continuous, then the function $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = f(g(x))$ is continuous.