Relations II

A relation $R \subseteq X \times X$ is *reflexive* if for all $x \in X$, xRx. It is *symmetric* if for all $(x, y) \in R$, yRx. It is *transitive* if xRy and yRz implies xRz.

1: Prove or disprove the following statements. If they are false, can you find additional conditions to make them true?

(a) If R is transitive and $x_1, x_2, \ldots, x_n$ are such that $x_i R x_{i+1}$ for $i = 1, \ldots, n-1$ (that is, $x_1 R x_2, x_2 R x_3, \ldots, x_{n-1} R x_n$), then $x_1 R x_n$.

(b) (Velleman pg. 187 n. 13) If R_1 and R_2 are symmetric, then $R_1 \cup R_2$ is symmetric.

(c) (Velleman pg. 189 n. 22) If R is symmetric and transitive, then R is reflexive.

2: Are the following relations reflexive? symmetric? transitive?

(a) In any X, xRy iff $x \neq y$.

(b) In $\mathbb{Z}$, nRm iff $n \mid m$.

(c) In $\mathbb{R}^2$, $\mathbf{x}R\mathbf{y}$ iff $\det\left(\begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix}\right) \geq 0$.

(d) For some X with a distinguished point $x \in X$, the relation on $\mathscr{P}(X)$ defined by ARB iff $x \in A \cap B$.

(e) For some X, the relation on $\mathscr{P}(X)$ defined by ARB iff $(A \cup B) \setminus (A \cap B) = \emptyset$.