Math 8
Worksheet 17

Functions I

A *function* $f : X \to Y$ is a relation from X to Y where each $x \in X$ has exactly one element $y \in Y$ such that $(x, y) \in f$. Usually, this is denoted $y = f(x)$.

1: Prove the following statements:

(a) (Velleman pg 234 n 11c) Suppose $f : A \to B$ and S is a relation on B. Define

$$R = \{(x, y) \in A : f(x) S f(y)\}.$$

Prove that if S is transitive, R is as well.

(b) Suppose R is a relation from X to Y. Prove that the relation f from X to $\mathscr{P}(Y)$ defined as

$$\{(x, B) : (\forall y \in Y)(xRy \iff y \in B)\}$$

is a function.

2: Are the following functions?

(a) $\{(M,d) \in M_n \times \mathbb{R} : \det(M) = d\}$

(b) $\left\{(x,y) \in \mathbb{R} \times \mathbb{R} : y = x^2\right\}$

(c) $\left\{(y,x) \in \mathbb{R} \times \mathbb{R} : y = x^2\right\}$

(d) $\left\{(y,x) \in \mathbb{R}^+ \times \mathbb{R} : y = x^2\right\}$

(e) $f : X \to X/R$ where $f(x) = [x]_R$ where R is an equivalence relation

(f) $f : X/R \to X$ where $f([x]_R) = x$ where R is an equivalence relation

(g) $f : X/R \to Y/S$ where R and S are equivalence relations, $g : X \to Y$ is a function, and $f([x]_R) = [g(x)]_S$