Logical Propositions and Connectives

Mathematical proofs involve statements (things which are either true or false) which are combined using logical connectives (such as “and,” “or,” or “not”) to form more complicated statements. As shorthand for representing arguments, we often use letters to stand for simple statements (such as P for “n is a prime number” or G for “$x > 0$”), and symbols for logical connectives ($\wedge$ for “and,” $\vee$ for “(inclusive) or,” and $\sim$ for “not”). A formula in these symbols is *well-formed* if it can be interpreted as a statement (regardless of whether the statement is true). A *proposition* is something which has a (definite, but not necessarily known) truth value.

1.1.1: Which of the following are propositions? Give the truth value for each proposition.

(a) What time is dinner?

(b) It is not the case that π is not a rational number.

(c) $x/2$ is a rational number.

(d) Either π is rational and 17 is prime, or $7 < 13$ and 81 is a perfect square.

(e) There are more than three false statements in the book, and this is one of them. [This question is taken from the book, and should be interpreted as a statement “in the book”.]

1.1.4/5: If P, Q, and R are true while S and K are false, which of the following are true? Consider this in the context of truth tables.

(a) $Q \wedge (R \wedge S)$

(b) $Q \vee (R \wedge S)$

(c) $(P \vee Q) \wedge (R \vee S)$

(d) $(\sim P \vee \sim Q) \vee (\sim R \vee \sim S)$

(e) $\sim P \vee \sim Q$

(f) $(\sim Q \vee S) \wedge (Q \vee S)$

(g) $(P \vee S) \wedge (P \vee K)$

(h) $K \wedge \sim (S \vee Q)$