Conditionals and Biconditionals

Most theorem statements rely on the conditional "if... then..." or "implies" and biconditional "if and only if..." or "is equivalent to...", denoted $\implies$ and $\iff$, respectively. The statement $P \implies Q$ is true if Q is true whenever P is (importantly, if P is known to be false, $P \implies Q$ is true regardless of whether Q is true or not), and $P \iff Q$ is true if P and Q share the same truth value (both true or both false).

1.2.5: Which of the following conditional sentences are true?

(a) If triangles have three sides, then squares have four sides?

(b) If hexagons have six sides, then the moon is made of cheese.

(c) If $7 + 6 = 14$, then $5 + 5 = 10$.

(d) The Nile River flows east only if 64 is a perfect square.

1.2.6: Which of the following are true?

(a) Triangles have three sides iff squares have four sides.

(b) $7 + 5 = 12$ if and only if $1 + 1 = 2$.

(c) $5 + 6 = 6 + 5$ iff $7 + 1 = 10$.

(d) A parallelogram has three sides iff 27 is prime.

1.2.13: Give, if possible, an example of a true conditional sentence for which

(a) the converse is true.

(b) the converse is false.

(c) the contrapositive is false.

(d) the contrapositive is true.

1.2.16: Determine whether each of the following is a tautology, a contradiction, or neither.

(a) $[(P \implies Q) \implies P] \implies P$

(b) $P \equiv P \wedge (P \vee Q)$

(c) $P \implies Q \iff P \wedge \sim Q$

(d) $P \implies [P \implies (P \implies Q)]$