Quantifiers

When referencing variables, we bind them with quantifiers. There are two basic quantifiers, the universal quantifier, usually read “for all” and written ∀, and the existential quantifier, usually read “there exists” and written ∃.

1.3.10: Which of the following are true in the universe of all real numbers?

(a) (∀x)(∃y)(x + y = 0)  
(b) (∃x)(∀y)(x + y = 0)  
(c) (∃x)(∃y)(x^2 + y^2 = −1)  
(d) (∀x)[x > 0 ⇒ (∃y)(y < 0 ∧ xy > 0)]  
(e) (∃y)(∃x)(∀z)(xy = xz)  
(f) (∃x)(∀y)(x ≤ y)
1.3.13: Which of the following are denials of \((\exists!x)P(x)\)?

(a) \((\forall x)P(x) \lor (\forall x)\sim P(x)\)  
(b) \((\forall x)\sim P(x) \lor (\exists y)(\exists z)(y \neq z \land P(y) \land P(z))\)

(c) \((\forall x)[P(x) \implies (\exists y)(P(y) \land x \neq y)]\)  
(d) \((\exists x)(\forall y)[(P(x) \land P(y)) \implies x = y]\)

1.3.3: Translate these definitions from the Appendix into quantified sentences.

(a) The natural number \(a\) divides the natural number \(b\).
(b) The natural number \(n\) is prime.
(c) The natural number \(n\) is composite.