Quantifiers

When referencing variables, we bind them with quantifiers. There are two basic quantifiers, the universal quantifier, usually read “for all” and written $\forall$, and the existential quantifier, usually read “there exists” and written $\exists$.

1.3.10: Which of the following are true in the universe of all real numbers?

(a) $(\forall x)(\exists y)(x + y = 0)$

(b) $(\exists x)(\forall y)(x + y = 0)$

(c) $(\exists x)(\exists y)(x^2 + y^2 = -1)$

(d) $(\forall x)[x > 0 \implies (\exists y)(y < 0 \wedge xy > 0)]$

(e) $(\forall y)(\exists x)(\forall z)(xy = xz)$

(f) $(\exists x)(\forall y)(x \leq y)$

1.3.13: Which of the following are denials of $(\exists! x)P(x)$?

(a) $(\forall x)P(x) \lor (\forall x) \sim P(x)$

(b) $(\forall x) \sim P(x) \lor (\exists y)(\exists z)(y \neq z \land P(y) \land P(z))$

(c) $(\forall x)[P(x) \implies (\exists y)(P(y) \land x \neq y)]$

(d) $\sim (\forall x)(\forall y)[(P(x) \land P(y)) \implies x = y]$

1.3.3: Translate these definitions from the *Appendix* into quantified sentences.

(a) The natural number *a divides* the natural number *b*.

(b) The natural number *n* is *prime*.

(c) The natural number *n* is *composite*.