Proofs II

1.6.3: Prove that if every even natural number greater than 2 is the sum of two primes, then every odd natural number greater than 5 is the sum of three primes.

1.6.4 Provide either a proof or a counterexample for each of these statements.

(a) For all positive integers x, $x^2 + x + 41$ is a prime.

(b) In the universe of all reals, $(\forall x)(\exists y)(x + y = 0)$.

(c) In the universe of all reals, $(\forall x)(\forall y)(x > 1 \wedge y > 0 \implies y^x > x)$.

(d) For integers a, b, c, if a divides bc, then either a divides b, or a divides c.

1.7.3: Prove that

(a) $5n^2 + 3n + 4$ is even, for all integers n.

(b) for all integers n, if $5n + 1$ is even, then $2n^2 + 3n + 4$ is odd.

(c) the sum of five consecutive integers is always dividible by 5.

(d) $n^3 - n$ is divisible by 6, for all intgers n.