Sets II

2.2.6: Give an example of nonempty sets A, B, and C such that:

(a) $C \subseteq A \cup B$ and $A \cap B \nsubseteq C$

(b)

(a) False: the set contains sets of numbers, but 5 is a number.

(b) True, but 6 is not.

(c) False: the set contains a number, but $\{5\}$ is a set of numbers.

(d) True.

(e) True.

(f) False.

2.1.6: Give an example, if there is one, of sets A, B, and C such that the following are true.

(a) $A \subseteq B$, $B \nsubseteq C$, and $A \subseteq C$.

(b) $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$.

(c) $A \nsubseteq B$, $B \nsubseteq C$, and $A \subseteq C$

(d) $A \subseteq B$, $B \nsubseteq C$, and $A \nsubseteq C$.

(e) $A \in B$, $A \subseteq B$, and $B \subseteq C$.

(a) Here we can take $A = \{a\}$, $B = \{a, b\}$, and $C = \{a, c\}$.

(b) Here we can take $A = B = C = \{a\}$.

(c) Here we can take $A = \{a\}$, $B = \{b\}$, and $C = \{a, c\}$.

(d) Here we can take $A = \{a\}$, $B = \{a, b\}$, and $C = \{c\}$.

(e) Here we can take $A = \{a\}$, $B = \{a, \{a\}\}$, and $C = \{a, \{a\}, \{\{a\}\}\}$.

(f) Finally, this is not possible: since $A \subset B$, if $x \in A$, then $x \in B$, and since $B \subset C$, if $x \in B$, then $x \in C$. So, if $x \in A$, then $x \in C$, so $A \subseteq C$ if $A \subseteq B$ and $B \subseteq C$.

2.1.17: True or false?

(a) $\emptyset \in \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(b) $\{\emptyset\} \in \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(c) $\{\{\emptyset\}\} \in \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(d) $\emptyset \subseteq \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(e) $\{\emptyset\} \subseteq \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(f) $\{\{\emptyset\}\} \subseteq \mathscr{P}(\{\emptyset, \{\emptyset\}\})$

(g) $3 \in \mathbb{Q}$

(h) $\{3\} \subseteq \mathscr{P}(\mathbb{Q})$

(i) $\{3\} \in \mathscr{P}(\mathbb{Q})$

(j) $\{\{3\}\} \subseteq \mathscr{P}(\mathbb{Q})$

(a) The power set

$$\mathscr{P}(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}.$$

So, this is true.

(b) This is also true.

(c) This is, yet again, true.

(d) This is true (the empty set is a subset of all sets).

(e) This is true becasue $\emptyset$ is in the power set.

(f) This is true because $\{\emptyset\}$ is in the power set.

(g) This is true, $3 = \frac{3}{1}$ is rational.

(h) This is false: 3 is not in the power set of rationals, whose elements are subsets of rationals.

(i) This is true.

(j) This is true (it means the same thing as the line above).