Sets III

2.1.18: Let A and B be sets. Prove that

(a) $A = B$ if and only if $\mathscr{P}(A) = \mathscr{P}(B)$.

(b) if A is a proper subset of B, then $\mathscr{P}(A)$ is a proper subset of $\mathscr{P}(B)$.

2.3.14: Let $\mathscr{A}$ be a family of pairwise disjoint sets. Prove that if $\mathscr{B} \subseteq \mathscr{A}$, then $\mathscr{B}$ must be a pairwise disjoint family of sets.

2.3.17: Suppose $\mathscr{A} = \{A_i : i \in \mathbb{N}\}$ is a family of sets such that for all $i, j \in \mathbb{N}$, if $i \leq j$, then $A_j \subseteq A_i$. (Such a family is called a *nested family* of sets.)

(a) Prove that for every $k \in \mathbb{N}$, $\bigcap_{i=1}^{k} A_i = A_k$.

2.4.5: Use the PMI to prove the following for all natural numbers:

(a) n^3+5n+6 is divisible by 3

(b) 4^n-1 is divisible by 3

(c) n^3-n is divisible by 6

(d) $(n^3-n)(n+2)$ is divisible by 12