Induction

2.5.7: Use the Principle of Complete Induction to prove the following properties of the Fibonacci numbers:

(a) f_n is a natural number for all natural numbers n.

(b) $f_{n+6} = 4f_{n+3} + f_n$ for all natural numbers n.

(c) For every natural number a, $f_a f_n + f_{a+1} f_{n+1} = f_{a+n+1}$ for all natural numbers n.

2.6.13: Prove the number of permutations of a subcollection of r objects from a larger collection of n objects is $\frac{n!}{(n-r)!}$

(b) by induction on n.

2.6.24: The nth *pyramid number*, p_n, is the number of balls of equal diameter that can be stacked in a pyramid whose base is an n by n square. The first few pyramid numbers are $p_1 = 1$, $p_2 = 5$, $p_3 = 14$, and $p_4 = 30$. Show that

(a) $p_n = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every natural number n.

(b) $p_n = \binom{n+2}{3} + \binom{n+1}{3}$ for $n \geq 2$.