Combinatorial Proofs

2.6.22:

(a) Give a combinatorial proof that if n is an odd integer, then the number of ways to select an even number of objects from a set of n objects is equal to the number of ways to select an odd number of objects.

(b) Give a combinatorial proof of Vandermonde's identity: For positive integers m and n and an integer r such that $0 \leq r \leq n+m$, $\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \binom{n}{2}\binom{m}{r-2} + \cdots + \binom{n}{r}\binom{m}{0}$.

(c) Prove that $\binom{2n}{n} + \binom{2n}{n+1} = \frac{1}{2}\binom{2n+2}{n+1}$.

2.6.23: Give a combinatorial argument that $n^2 = 2\binom{n}{2} + n$.