Math 4A
Worksheet 1.6

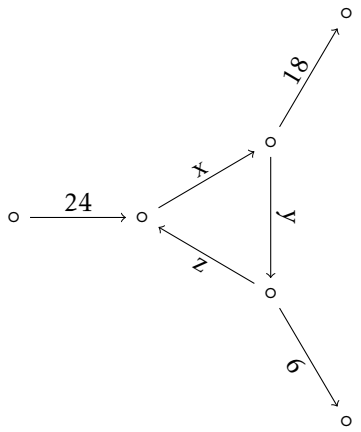
Applications of Linear Systems

The Punch Line: Linear systems of equations can describe many interesting situations.

Set-Up: In a situation you can model with linear equations, there will be a number of *constraints*: things which must be equal because of the laws governing what's going on (e.g., laws of physics, economic principles, or definitions of quantities and the values you observe for them). These will give you the equations that you can solve to get information about the variables you care about

1 In the past three men's soccer games, the Gauchos averaged $\frac{5}{3}$ goals per game. They scored the same number of goals in the most recent two games, but three games ago they scored an additional two goals. How many points did they score in each game?

2 Suppose you're watching a bike loop on campus and writing down the net number of bicycles travelling through each part of the loop (the number of bikes going one direction minus the number going the other direction). You're able to observe how many net bikes per minute enter and leave through each of the three spokes, but aren't able to count well inside the loop. Luckily, you can use linear algebra to learn about how many net bikes per minute travel through each part of the loop (which is to say, find all solutions for x, y, and z that are consistent with the rest of the information about the problem)!



3 (Example 1 in Section 1.6) In an *exchange model* of economics, an economy is divided into different sectors which depend on each others' products to produce their output. Suppose we know for each sector its total output for one year and exactly how this output is divided or "exchanged" among the other sectors of the economy. The total dollar (or other monetary unit) value of each sector's output is called the *price* of that output. There is an *equilibrium price* for this kind of model, where each sectors income exactly balances its expenses. We wish to find this equilibrium.

Suppose we have an economy described by the following table:

Distribution of output from:			
Coal	**Electric**	**Steel**	**Purchased by:**
0.0	0.4	0.6	Coal
0.6	0.1	0.2	Electric
0.4	0.5	0.2	Steel

If we denote the price of the total annual outputs of the Coal, Electric, and Steel sectors by p_C, p_E, and p_S respectively, what is the equilibrium price (or describe them if there is more than one).

Under the Hood: When can we use linear equations to model something? The basic setup of a linear system involves a collection of quantities that we know are equal to known values (or each other), and a collection of variables. We can use a linear system when the way the quantities depend on changes to the variables is independent of the actual values of the variables (adding the same amount to a variable changes each quantity in the same way, no matter what value any of the variables have).