Math 4A
Worksheet 2.1

Matrix Operations

The Punch Line: Various operations combining linear transformations can be realized with some standard matrix operations.

Addition and Scalar Multiplication: Just like with vector operations, the sum of matrices and the multiplication by a *scalar* (just a number, as opposed to a vector or matrix) are done component-by-component.

1 Try the following matrix operations:

(a) $3\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Matrix Multiplication: To multiply two matrices, we create a new matrix, each of whose columns is the result of the matrix-vector product of the left matrix with the corresponding column of the right matrix (the product will have the same number of rows as the left matrix, and the same number of columns as the right matrix). To get the ij entry (ith row and jth column) we could multiply the ith row of the left matrix with the jth column of the right matrix.

2 Multiply these matrices (if possible, otherwise say why it isn't):

(a) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}$

Transpose: The last matrix operation for today is the *transpose*, where you switch the roles of rows and columns. That is, if you get an $n \times m$ matrix, its transpose will be $m \times n$.

3 Compute the following operations for the matrices given:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix}.$$

(a) A^T

(b) B^T

(c) C^T

(d) $(BA)^T$

(e) $A^T B^T$

(f) $(BAC)^T$

(g) AA^T

(h) $A^T A$

(i) $(AA^T - B)^T$

What do these operations mean? Matrix addition and scalar multiplication correspond to adding and scaling the results of applying the linear transformation of the matrix, respectively. Matrix multiplication corresponds to composing the two linear transformations (applying one to the result of another). Transposition is a little weirder, and corresponds to switching the roles of variables and coefficients in a linear equation.