

Transformational Geometry

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Geo-Board Activity

Reproduce the set of figures on the geo-board and on the geo-board sheet. Respond to the following in your notes.

- 1 Write a brief description about what is happening from the blue figure to the red figure.
- 2 Make a list of words that your students might use to describe this concept.

Definitions

Transformation

A *transformation* of the plane is a one-to-one mapping of the plane onto itself.

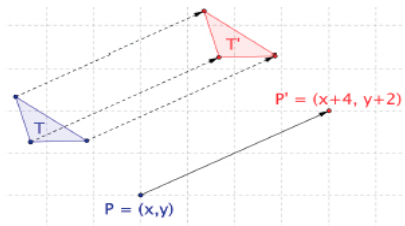
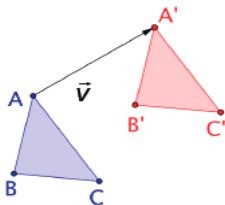
- Mapping
- One-to-one
- Onto

Image

The *image* of a set of points is the resulting set of points under a mapping.

Translation

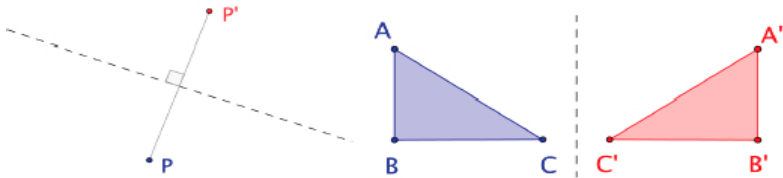
A *translation* of the plane is a transformation which shifts all points on the plane in the same direction and in the same distance. That is, given a vector \vec{v} , the image P' of a point P is the point for which $\overline{PP'} \parallel \vec{v}$ and $PP' = |\vec{v}|$.



Reflection

A *reflection* about an axis is a transformation which maps every point P to a point P' such that

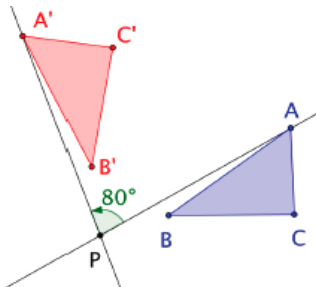
- 1 If P is not on the axis of reflection, then the axis is the perpendicular bisector of $\overline{PP'}$, and
- 2 If P is on the axis of reflection, then $P' = P$.



Rotation

A *rotation* about a point P through angle α is a transformation such that

- 1 If A is different from P , then $PA = PA'$ and the measure of $\angle APA' = \alpha$, and
- 2 If A is the same as P , then $A' = A$.

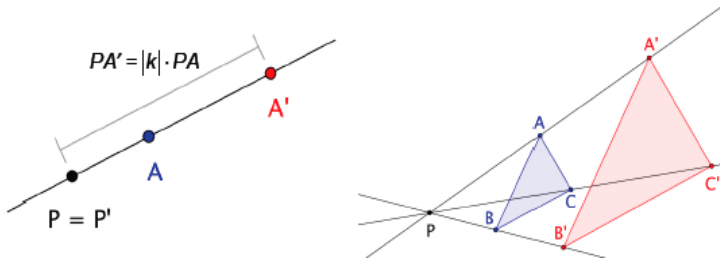


Dilation

A *dilation* with center P and scale factor $k \neq 0$ is a transformation such that

- ① If A is different from P , then the image A' lies on \overleftrightarrow{PA} and $PA' = |k| \cdot PA$, and
- ② If A is the same as P , then $A' = A$.

Note: If $k < 0$, then P will lie between A and A' .



On the geo-board, come up with your own examples of transformations. Record your examples on the geo-board sheets. In each case, write a brief description in your notes using the vocabulary we have discussed.

- Translation
- Reflection
- Rotation
- Dilation

Isometry

An *isometry* is a transformation that preserves length.

(Also called *rigid transformations*.)

The following types of translations are isometries:

- Translation
- Reflection
- Rotation

A dilation is not an isometry unless $k = \pm 1$.

(Dilation is also called *non-rigid transformation*.)

Check for understanding:

On your geo-board display one isometry. Reproduce the isometry on the geo-board sheet, and describe it in your notes using the proper vocabulary.

Composition

A *composition* of transformations occurs when a transformation is applied to the image of a set of points from another transformation.

On your geo-board show an example of a composition of isometries. Reproduce the composition on the geo-board sheet, and describe it in your notes using the proper vocabulary.

Try one or two additional examples.

Commutative Property: Order does not effect the result.
(Commutative Property for Multiplication: $a \cdot b = b \cdot a$.)

Experiment further with different transformation compositions.
Make a conjecture about whether or not the commutative property holds for transformation composition.
Prove your conjecture.

Define congruence.

Congruence

A figure in the plane is *congruent* to another figure if the second can be obtained from the first by a sequence of translations, rotations and reflections (by a sequence of isometries).

Given two congruent figures, describe a sequence that exhibits the congruence between them.

Activity: Introduction to Dilations

On your geo-board show an example of a composition of one or more isometries with a dilation. Reproduce the composition on the geo-board sheet, and describe it in your notes using the proper vocabulary.

Define similarity.

Similarity

A figure in the plane is *similar* to another figure if the second can be obtained from the first by a sequence translations, rotations, reflections, and dilations.

Given two similar figures, describe a sequence that exhibits the similarity between them.

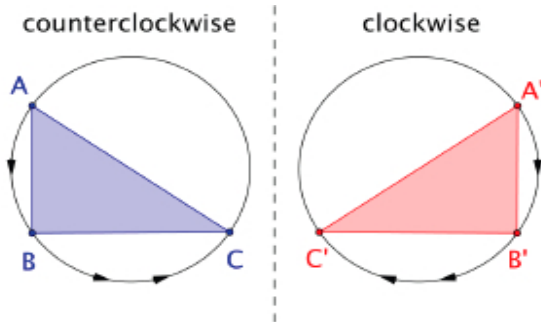
Properties

We want to know if a transformation

- preserves distance. Lines are taken to lines, and line segments to line segments of the same length.
- preserves parallelism. Parallel lines are taken to parallel lines.
- preserves angle measure. Angles are taken to angles of the same measure.
- preserves collinearity. If three points lie on the same line, then their images lie on the same line.
- preserves betweenness. If B is between A and C on a line, then B' is between A' and C' on the images.
- preserves orientation. The points A , B and C have the same orientation as their corresponding points A' , B' and C' on the image.

Orientation

Orientation refers to the arrangement of points relative to one another in a figure. If points A , B and C lie on a circle, then we say $\triangle ABC$ has a clockwise orientation if the path on the circle from A through B to C traverses the circle in a clockwise direction. Similarly for counterclockwise.



An Introduction to Transformations Handout

Geometric Transformations Workout

Geometry

8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
 - a. Lines are taken to lines, and line segments to line segments of the same length.
 - b. Angles are taken to angles of the same measure.
 - c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Congruence

G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Similarity, Right Triangles, and Trigonometry

G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
 - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
 - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Coordinate Geometry