Math 1c TA: Padraic Bartlett

# Homework 2: The Derivative in $\mathbb{R}^n$

*Week 2* *Caltech 2013*

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

#2.3.3(a), (c). Find the two partial derivatives $\partial w/\partial x$ and $\partial w/\partial y$:

(a) $w = xe^{x^2+y^2}$

(c) $w = e^{xy}\ln(x^2+y^2)$

#2.3.10(a), (c). Compute the matrix of partial derivatives of the following two functions:

(a) $f(x,y) = (e^x, \sin(xy))$

(c) $f(x,y) = (x+y, x-y, xy)$

#2.5.8. Let $f(u,v,w) = (e^{u-w}, \cos(u+v)+\sin(u+v+w))$, and $g(x,y) = (e^x, \cos(y-x), e^{-y})$. Calculate $f \circ g$ and $\mathbf{D}(f \circ g)(0,0)$.

#2.5.9. Find $(\partial/\partial s)(f \circ T)(1,0)$, where $f(u,v) = \cos(u)\sin(v)$ and $T(s,t) = (\cos(t^2 s), \ln(\sqrt{1+s^2}))$.

#2.6.10(a). Compute the gradient $\nabla f$ of the function $f(x,y,z) = 1/\sqrt{x^2+y^2+z^2}$.

#2.6.18(a). Compute the directional derivative of the function $f(x,y,z) = xy^2 + y^2z^3 + z^3x$ at the point $P = (4,-2,-1)$, in the direction $\mathbf{v} = \frac{1}{\sqrt{14}}(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$.

#3.1.4. Compute all of the second partial derivatives $\partial^2 f/\partial x^2, \partial^2 f/\partial x \partial y, \partial^2 f/\partial y \partial x, \partial^2 f/\partial y^2$ of the function $f(x,y) = e^{-xy^2} + y^3x^4$. Verify that the mixed partials of this function are in fact the same.

#3.1.6. Compute all of the second partial derivatives $\partial^2 f/\partial x^2, \partial^2 f/\partial x \partial y, \partial^2 f/\partial y \partial x, \partial^2 f/\partial y^2$ of the function $f(x,y) = \log(x-y)$. Again, verify that the mixed partials of this function are in fact the same.