Math 1c	TA:

Homework 3: Applications of the Derivative in $\mathbb{R}^n$

Week 3 — *Caltech 2013*

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

As always, show your work. In particular, when classifying critical points, don't just say "blah is a max, foo is a min"; instead, explain your reasoning.

#3.2.5. Find the second-order Taylor approximation for the function $f(x, y) = e^{x+y}$ around the point $(0, 0)$.

#3.2.10. Find the second-order Taylor approximation for the function $f(x, y) = x\cos(\pi y) - y\sin(\pi x)$ around the point $(1, 2)$.

#3.3.4 Find the critical points of the function $f(x, y) = x^2 + y^2 + 3xy$. Classify these critical points into local minima, maxima, and saddle points.

#3.3.9. Show that $(0, 0)$, $(\sqrt{\pi/2}, \sqrt{\pi/2})$, and$(0, \sqrt{\pi})$ are critical points of the function $f(x, y) = \cos(x^2 + y^2)$. Classify these critical points.

#3.3.17 Find all local extrema of the function $f(x, y) = 8y^3 + 12x^2 - 24xy$.

#3.3.35 Find the critical points of the function $f(x, y, z) = x^2 + y^2 + z^2 + xy$. Classify these critical points into local minima, maxima, and saddle points.

#3.4.3 Find the extrema of the function $f(x, y, z) = x - y + z$ subject to the constraint $x^2 + y^2 + z^2 = 2$.

#3.4.7 Find the extrema of the function $f(x, y) = 3x+2y$ subject to the constraint $2x^2+3y^2 = 3$.

#3.4.13 Consider the function $f(x, y) = x^2 + xy + y^2$. Use the method of Lagrange multipliers to find the minimum and maximum values of f on the unit circle $S = \{(x, y) : x^2+y^2 = 1\}$. Use this information to determine the absolute maximum and minimum values of f on the unit disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$.

#3.4.23 Find the absolute maximum and minimum values of the function $f(x, y, z) = x+y-z$ on the ball $B = \{(x, y, z) = x^2 + y^2 + z^2 \leq 1\}$.