

Homework 6: The Integral in \mathbb{R}^n

For details on the collaboration policy, due dates, etc., please refer to [the Malc course webpage](#). If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

As always, show your work.

#5.2.4. Suppose that R is the region $[0, 1] \times [-2, 2]$. Evaluate the integral

$$\iint_R \frac{y}{1+x^2} dx dy.$$

#5.3.4(d). Evaluate the following integral. Additionally, sketch the region of \mathbb{R}^2 that this integral is being calculated over.

$$\int_0^{\pi/2} \int_0^{\cos(x)} y \sin(x) dy dx$$

#5.3.11. Let D be the region $\{(x, y) : 1 \leq x^2 + y^2 \leq 2, y \geq 0\}$. Is D an elementary region? Evaluate the integral

$$\iint_D (1 + xy) dA.$$

#5.4.2. By changing the order of integration, evaluate the integral

$$\int_0^1 \int_y^1 \sin(x^2) dx dy.$$

#5.4.5. By changing the order of integration, evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy.$$

#5.5.4. Suppose that B is the box $[0, 1]^3$. Evaluate the integral

$$\iiint_B e^{-xy} y dx dy dz.$$

#5.5.12. Find the volume of the region bounded by the three surfaces $x^2 + 2y^2 = 2$, $z = 0$, and $x + y + 2z = 2$.

#5.5.18. Suppose that W is the region bounded by the planes $x = 0, y = 0, z = 0, z = 1$, and the cylinder $x^2 + y^2 = 1, x \geq 0, y \geq 0$. Evaluate the integral

$$\iiint_W z \, dx dy dz.$$

#6.2.11. Suppose that D is the disk $D = \{(x, y) : x^2 + y^2 \leq 4\}$. Evaluate the integral

$$\iint_D (x^2 + y^2)^{3/2} \, dx dy$$

#6.2.29. Suppose that a, b are a pair of positive real numbers such that $b < a$. Let W be the solid bounded by the two spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$. Evaluate the integral

$$\iiint_W \sqrt{x^2 + y^2 + z^2} e^{-x^2 - y^2 - z^2} \, dx dy dz.$$