Math 1c TA:

Homework 7: Surfaces and the Integral in $\mathbb{R}^n$

Week 7 *Caltech 2013*

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

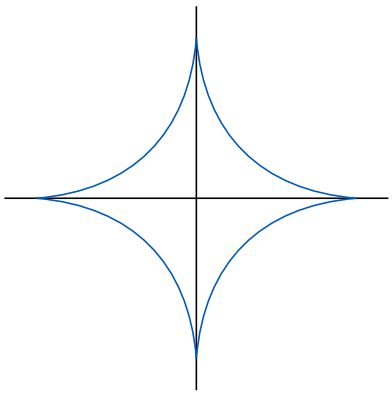
As always, show your work.

#7.1.19(a). Let $\mathbf{c}(t) = \left(t^2, t, 3\right)$, for $t \in [0,1]$. Find the length of the path given by $\mathbf{c}$.

#7.1.24. Compute the path integral of $f(x,y) = y^2$ over the graph of $y = e^x, x \in [0,1]$.

#7.2.8. Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x,y,z) = (y, 2x, y)$ and the path $\mathbf{c}$ is defined by the equation $\mathbf{c}(t) = (t, t^2, t^3), t \in [0,1]$.

#7.2.11. The image of the path $\mathbf{c}(t) = (\cos^3(t), \sin^3(t)), t \in [0, 2\pi]$ in the plane is illustrated below. Evaluate the integral of the vector field $\mathbf{F}(x,y) = (x,y)$ around this curve.



#7.3.14. Find the equation for the plane tangent to the surface $x = u^2, y = v^2, z = u^2 + v^2$ at the point $(u,v) = (1,1)$.

#7.4.6. Find the area of the surface defined by the two formulae $z = xy$ and $x^2 + y^2 \leq 2$.

#7.5.7. Compute the integral

$$\iint_S xy \, dS,$$

where S is the surface of the tetrahedron with sides $y = 0, z = 0, x + z = 1$, and $x = y$.

#7.5.19. Find the average value of $f(x,y,z) = x + z^2$ on the truncated cone $z^2 = x^2 + y^2$, with $z \in [1,4]$.

#7.6.4. Let $\mathbf{F}(x, y, z) = (2x, -2y, z^2)$. Evaluate the integral

$$\iint_S \mathbf{F} \cdot dS,$$

where S is the cylinder $x^2 + y^2 = 4$, $0 \leq z \leq 1$.

#7.6.19. Let the velocity field of a fluid be described by the vector field $\mathbf{F}(x, y, z) = (1, x, z)$. Compute how many cubic meters per second are crossing the surface described by the equations $x^2 + y^2 + z^2 = 1, z \geq 0$.