Math 1c	TA: Padraic Bartlett

Homework 7: Green's Theorem

Week 8 | *Caltech 2013*

For details on the collaboration policy, due dates, etc., please refer to the Ma1c course webpage. If you have any questions when working on the HW, please don't hesitate to contact your TA (or really any of the TA's,) or indeed even your fellow students!

As always, show your work.

#8.1.3. Suppose that D is the region $[-1,1] \times [-1,1]$, $P(x,y) = -y$, and $Q(x,y) = x$. Explicitly calculate $\int_{\partial D^+} Pdx + Qdy$ and $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$ individually. Verify that Green's theorem holds in this situation: i.e. that these two integrals are equal.

#8.1.4. Repeat #8.1.3 with $D = [-1,1] \times [-1,1]$, $P(x,y) = x$, $Q(x,y) = y$.

#8.1.11(a). Repeat #8.1.3 with $D = \{(x,y) : x^2 + y^2 \leq R^2\}$, $P(x,y) = xy^2$, $Q(x,y) = -yx^2$.

#8.1.11(c). Repeat #8.1.3 with $D = \{(x,y) : x^2 + y^2 \leq R^2\}$, $P(x,y) = xy$, $Q(x,y) = xy$.

#8.1.14(b). Suppose that D, P, Q satisfy the hypotheses of Green's theorem. Prove the following equality:

$$\int\limits_{\partial D^+} \left(Q\frac{\partial P}{\partial x} - P\frac{\partial Q}{\partial x}\right) dx + \left(P\frac{\partial Q}{\partial y} - Q\frac{\partial P}{\partial y}\right) dy = 2\iint\limits_D \left(P\frac{\partial^2 Q}{\partial x \partial y} - Q\frac{\partial^2 P}{\partial x \partial y}\right) dxdy.$$

#8.1.17. For any pair of arbitrary real numbers $0 < a < b$, repeat #8.1.3 with $D = \{(x,y) : a \leq x^2 + y^2 \leq b\}$, $P(x,y) = 2x^3 - y^3$, $Q(x,y) = x^3 + y^3$. Note that D's boundary consists of two distinct circles, with the inner circle oriented in the clockwise direction and the outer oriented in the counterclockwise direction.

#8.1.19(a). Verify the divergence theorem for $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ and $D = \{(x,y) : x^2 + y^2 \leq 1\}$. In other words, show that $\int_{\partial D} \mathbf{F} \cdot \mathbf{n}\, ds$ is equal to $\iint_D \text{div } F dA$.

#8.1.20. Let D denote the unit disk, $P(x,y) = -\dfrac{y}{x^2+y^2}$, and $Q(x,y) = \dfrac{x}{x^2+y^2}$. Show that the claim in Green's theorem does not hold here: i.e. $\int_{\partial D^+} Pdx + Qdy$ is not equal to $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$. Explain why.