

MATH 1D, HW #1 – QUESTIONS

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Instructions: Choose **three** questions out of the **four** below to complete! Also, justify everything you claim. These are difficult questions! Write me if you have any questions.

Question 0.1. Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive numbers, chosen so that the sums $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ both diverge. Does the sum

$$\sum_{n=1}^{\infty} \frac{1}{a_n + b_n}$$

also have to diverge?

Question 0.2. Does the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

converge?

Question 0.3. Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive numbers, with the property that $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ both converge. Does the sum

$$\sum_{n=1}^{\infty} a_n b_n$$

also have to converge?

Question 0.4. Suppose that $\{a_n\}$ is a sequence such that

- $\lim_{n \rightarrow \infty} a_n = 0$, and
- the partial sums of the a_n 's are bounded by ± 1 — in other words,

$$-1 \leq \sum_{k=1}^n a_k \leq 1,$$

for all n .

Does the sum

$$\sum_{n=1}^{\infty} a_n$$

have to converge?