MATH 1D, HW #4 – QUESTIONS

INSTRUCTOR: PADRAIC BARTLETT

Instructions: Choose **three** questions out of the **four** below to complete! Also, justify everything you claim. Many of these are difficult questions! Write me if you have any questions.

Question 0.1. Find the Taylor series of the function $\frac{1}{x-a}$, and use it to find the Taylor series of the function $\log(x-a)$.

Question 0.2. Suppose that the power series $f(x) = \sum_{n=0}^{\infty} a_n x^n$ converges on the interval $(-1,1)$; furthermore, suppose that $f(x) = 0$ on all of $(-1,1)$. Show that $a_n = 0$ for every $n \in \mathbb{N}$.

Question 0.3. Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is an even function. Show that $a_n = 0$ whenever n is odd.

Similarly, if $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is an odd function, show that $a_n = 0$ whenever n is even.

Question 0.4. So: on Tuesday, the following curious result was stated: if we let $\{n_k\}$ be the sequence consisting of all natural numbers that do not have a 9 anywhere in their digits, then

$$\sum_{k=1}^{\infty} \frac{1}{n_k} < 80,$$

and thus converges. Prove this.

(Hint: how many n-digit numbers don't have a 9 in them? Given this, come up with a bound on the size of any n-digit number, and use these two pieces of information to bound the above series.)

Date: Due Date: Thursday, Feb. 11, at 4 p.m.