| Math 1d | Instructor: Padraic Bartlett |
|---|---|

# Homework 4

*Week 5* — *Caltech - Winter 2012*

**Instructions**: Choose **three** questions out of the following **six** to complete. If you attempt more than three questions, your three highest scores will be recorded as your grade. Also, write me/Daiqi an email if you get stuck!

Resources allowed: Wikipedia, Mathematica, your notes, the online class notes, textbooks, and your classmates. If you've completed a problem via Mathematica or writing a program, attach your code to receive credit.

Also: I put on a 7th problem. If you want, instead of doing this set, you can simply solve problem 7! Please email Daiqi and I your code/a compiled program if you do this.

**Standard Exercises**:

1. For each of the following power series, find the sets on which they converge:

$$\text{(a)} \qquad \sum_{n=1}^{\infty} x^n \cdot n^n.$$

$$\text{(b)} \qquad \sum_{n=2}^{\infty} \frac{x^n}{n \cdot \ln(n)}.$$

$$\text{(c)} \qquad \sum_{n=2}^{\infty} \frac{x^n}{(\ln(n))^n}.$$

2. We mentioned the following orthogonality relations in class on Wednesday when we were talking about Fourier series:

$$\int_{-\pi}^{\pi} \sin^2(nx)dx = \pi, \forall n \in \mathbb{N}.$$

$$\int_{-\pi}^{\pi} \cos^2(nx)dx = \pi, \forall n \in \mathbb{N}.$$

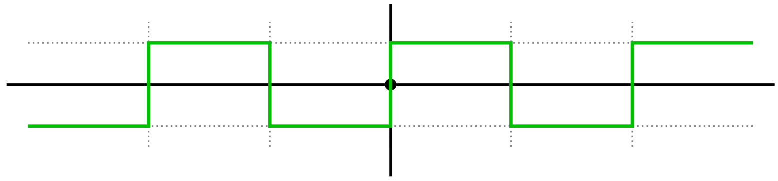
$$\int_{-\pi}^{\pi} \sin(nx)\sin(mx)dx = 0, \forall n \neq m \in \mathbb{N}.$$

$$\int_{-\pi}^{\pi} \cos(nx)\cos(mx)dx = 0, \forall n \neq m \in \mathbb{N}.$$

$$\int_{-\pi}^{\pi} \sin(nx)\cos(mx)dx = 0, \forall n, m \in \mathbb{N}.$$

Prove any **two** of these relations. (Calculating all of these integrals involves pretty much the same set of tricks, which is why we're not asking you to show *all* of them.)

3. (a) Find the Fourier series of the **square wave** $s(x)$:



$$s(x) = \begin{cases} -1, & x \in [-\pi, 0), \\ 1, & x \in [0, \pi), \\ s(x \pm 2\pi), & \forall x \in \mathbb{R}. \end{cases}$$

(b) Using (Mathematica/Maple/Matlab/Wolfram Alpha/your favorite program), graph the sum of the first hundred terms of the Fourier series from $-\pi$ to $\pi$, and attach your graph. Does it look like a square wave? What does it look like is occuring near the "jump discontinuities" at $-\pi, 0$, and $\pi$? (This doesn't have to be rigorous: just describe visually what you see. If you **do** want a rigorous discussion of what's going on here, look up the Gibbs phenomenon on Wikipedia.)

**More Interesting Problems**:

4. Using Mathematica's Play function and the harmonic analysis at http://hyperphysics.phy-astr.gsu.edu/hbase/music/flutew.html, create a Fourier series that sounds like a flute playing $F4$ when played in Mathematica. (If you are unsure what this means, ask me!)

5. Suppose that $P(x) = \sum_{n=0}^{\infty} a_n x^n$ is a power series such that $p(x) = 0, \forall x$. Show that $a_n = 0$, for all $n$.

   (Hint: First, show that $\frac{d^n}{dx^n}(p(x))\Big|_0 = n! \cdot a_n$. Then, show that all of $p(x)$'s derivatives at 0 are 0. )

6. (a) Suppose that $P(x) = \sum_{n=0}^{\infty} a_n x^n$ is an even[1] function. Show that $a_n$ is 0 whenever $n$ is odd.

   (b) Suppose that $P(x) = \sum_{n=0}^{\infty} a_n x^n$ is an odd[2] function. Show that $a_n$ is 0 whenever $n$ is even.

   (Hint: Use problem (5). You are welcome to use problem (5) even if you haven't proven it.)

**Far More Interesting Problems**:

7. Create a synthesizer. Specifically, create a program that when ran does the following:

   - First, asks you for your choice of instrument (provide at least two choices of instrument, like sine wave, sawtooth wave, square wave, clarinet-ish wave.)
   - Then, it takes in a number 1 through 12, interprets such a number as one of the 12 musical notes $A, B\flat, B, \ldots G\sharp$ ,and plays over speakers a Fourier series corresponding to this frequency and the chosen instrument.

   If you are unsure as to what counts as a "program," write me and I can explain! Also, if you do this, send both Daiqi and I a copy of your code, so we can play around with it.

[1] I.e. $P(x) = P(-x)$.

[2] I.e. $P(x) = -P(-x)$.