Generating Functions | Instructor: Paddy

Homework 1: An Introduction to Generating Functions

Week 2 | *Mathcamp 2010*

Do as many of these questions as you feel you need to become comfortable with the material! Especially with this first set; there's a lot of work here, but I'm not expecting anyone to do all of it.

1. Find a simple closed form for the ordinary generating functions of the following sequences:
   (a) $\{n\}_{n=0}^{\infty}$
   (b) $\{an+b\}_{n=0}^{\infty}$
   (c) $\{2^n\}_{n=0}^{\infty}$
   (d) $\{a_n\}_{n=0}^{\infty}$, with the recurrence relation $a_{n+1} = 2a_n + 1, a_0 = 0$.
   (e) $\{a_n\}_{n=0}^{\infty}$, with the recurrence relation $a_{n+1} = 2a_n - a_{n-1}, a_0 = 0, a_1 = 1$.
2. Find a simple closed form for the exponential generating functions of the following sequences:
   (a) $\{n!\}_{n=0}^{\infty}$
   (b) $\{2^n\}_{n=0}^{\infty}$
   (c) $\{a_n\}_{n=0}^{\infty}$, with the recurrence relation $a_{n+1} = (n-1)a_{n-1}, a_0 = a_1 = 1$.
3. Find the coefficient of x^n in the power series representation of
$$\frac{1}{(1-x^3)^2}$$
4. Let a_n be the number of subsets of $[n] = \{0, 1, \ldots, n\}$ that do not contain any pairs of consecutive elements.
   (a) Find a recurrence relation for a_n.
   (b) Use this to find a closed form for the ordinary generating function of $\{a_n\}_{n=0}^{\infty}$.
   (c) Use this to find an explicit formula for a_n.