Generating Functions	Instructor: Paddy

Homework 2: Multivariable Generating Functions

Week 2	*Mathcamp 2010*

Do as many of these questions as you want!

1. Show that the ordinary generating function for the sequence $a_n = \binom{2n}{n}$ is $(1-4x)^{-1/2}$.

2. Let $f(m,n)$ denote the number of increasing paths from $(0,0)$ to (m,n) along the integer lattice $\mathbb{N} \times \mathbb{N}$. Find a closed form for the ordinary generating function

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} f(n,m) x^n y^m,$$

and use this to find a closed form for $f(n,m)$.

3. Let $a_n = f(n,n)$. Find the ordinary generating function of the a_n's.

4. We claimed, at the end of class, that

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \sum_{r=1}^{k} (-1)^{k-r} \cdot \frac{r^n}{r! \cdot (k-r)!}.$$

Prove this.

5. Show that

$$\sum_{k=0}^{\infty} \left\{ \begin{matrix} n \\ k \end{matrix} \right\} y^k = \left(y + y \cdot \frac{d}{dy} \right)^n \circ 1,$$

where $\frac{d}{dy}$ denotes the differentiation operator. Can you get any more information from this?