Generating Functions | Instructor: Paddy

Homework 4: Sieves

Week 2 | *Mathcamp 2010*

Do as many as you want! They are a bit harder this time.

1. For positive integers n, k and r, how many of the permutations of $\{1, 2, \ldots n\}$ have exactly r cycles of length k?

2. Use the method of sieves to rederive the Stirling numbers of the second kind.

3. Find, in terms of $N(x)$, the ordinary generating function for the sequence $\{l_k\}_{k=0}^{\infty}$, where l_k counts the number of objects that have **at most** l properties.

4. Given a fixed n, find the number of permutations of $\{1, \ldots, n\}$ that consist of a single cycle $(a_1, a_2, \ldots a_n)$ for which $a_{n+1} \neq a_n + 1$. (i.e. for $n = 4$, there's exactly 1 such permutation: $(1, 4, 3, 2)$).