Probabilistic Methods in Graph Theory — Instructor: Paddy

Homework 1: An Introduction to the Probabilistic Method

Week 1 of 1 — *Mathcamp 2010*

1. Show that there is a 2-coloring of K_n with at most $\binom{n}{a} \cdot \left(2^{1-\binom{a}{2}}\right)$-many monochromatic K_a's in it.

2. Show that there is a 2-coloring of $K_{m,n}$ with at most $\binom{m}{a}\binom{n}{b} \cdot \left(2^{1-ab}\right)$-many monochromatic $K_{a,b}$'s in it.

3. Show that every set of $B = \{b_1, \ldots b_n\}$ of n nonzero integers contains a sum-free[1] subset of size $\geq n/3$.

4. Let G be a graph on at least 10 vertices, and suppose that G has the following property: if we add to G any edge not in G, then the number of copies of K_{10} in G increases. Show that $|G| \geq 8n - 36$.

[1] A subset of $\mathbb{R}$ is called sum-free if adding any two elements in the subset will never give you an element of the subset.