Probabilistic Methods in Graph Theory | Instructor: Paddy

Homework 3: Infinite Graphs

Week 1 of 1 | *Mathcamp 2010*

1. Show that deleting a finite number of edges or vertices from the Rado graph still leaves a graph that's isomorphic to the Rado graph.

2. Suppose you take the Rado graph and partition its vertices into k different sets $U_1, \ldots U_k$. Show that there is at least one k so that the induced subgraph on U_k is isomorphic to the Rado graph.

3. Consider the following graph: let $\mathbb{N}$ be our collection of vertices, and draw an edge $\{x, y\}$ whenever the x-th bit of y's binary representation is 1, or the y-th bit of x's binary representation is nonzero.

4. Show that the Rado graph is vertex-transitive[1].

5. Show that the Rado graph is homogeneous[2]

[1] A graph is vertex-transitive iff for any two vertices x and y, there is an automorphism of the graph sending x to y. An automorphism is an isomorphism of a graph with itself.

[2] A graph G is homogeneous iff any isomorphism between two finite induced subgraphs of G can be extended to an automorphism of all of G.