

Homework 4: Infinte Graphs

*Week 2**Mathcamp 2010*

1. In the random graph $G_{n,1/(2n)}$, show that the probability that G will have connected components of size larger than $n/4$ goes to 0 as n goes to infinity.
2. In the random graph $G_{n,1/n}$, show that the probability that G will have a connected components of size larger than \sqrt{n} goes to 1 as n goes to infinity.
3. Show that if $p > \frac{2\ln(n)}{n}$, then a random graph $G = G_{n,p}$ is connected with probability 1.
4. Prove the following claim: for every constant $p \in (0, 1)$ and $\epsilon > 0$, the probability that a random graph $G = G_{n,p}$ has

$$\chi(G) > \frac{\log(1/q)}{2 + \epsilon} \cdot \frac{n}{\log(n)}$$

goes to 1 as n goes to infinity.