Latin Squares | Instructor: Paddy

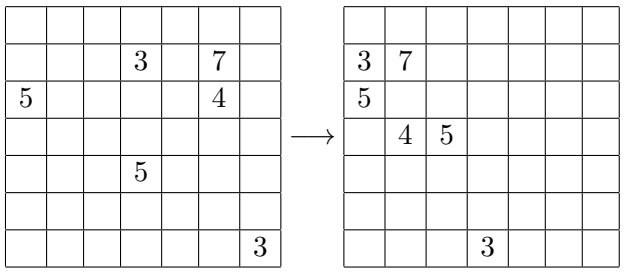
Homework 1: Latin Squares!

Week 4 | *Mathcamp 2010*

1. Prove the following lemma:

   Lemma 1 *If P is a partial latin square with $\leq n-1$ entries and $> n/2$ distinct symbols used in those entries, we can find an equivalent latin square P' with the following properties:*

   - *There is exactly one cell with symbol n; the rest all have symbols $\leq n-1$.*
   - *This n-symbol lies on the main diagonal of our matrix: the rest lie strictly beneath the main diagonal.*



2. Find 4 mutually orthogonal 5×5 latin squares; in general, show how we can find $n-1$ mutually orthogonal $n \times n$ latin squares, whenever n is a prime.

3. Find a 4×4 latin square that has no orthogonal mate.

4. In the second problem set, we showed that a $n \times n$ latin square L is equivalent to a 1-factorization of $K_{n,n}$. Show that a latin square L has an orthogonal mate if there is a coloring of L's associated 1-factorization in which each 1-factor contains an edge of every color.

5. (For those people who attended Yuval's talk) Find a quandle whose multiplication table is a latin square.