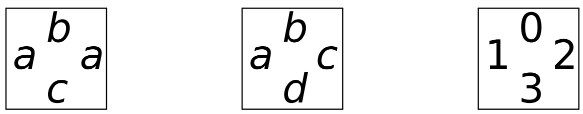
The Unit Distance Graph | Instructor: Paddy

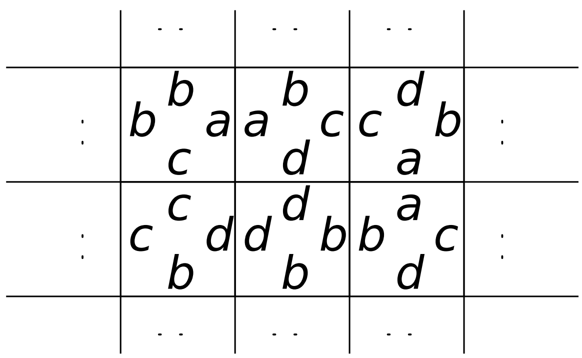
Homework 1: The Unit Distance Graph Problem; König's Lemma

Week 4 | *Mathcamp 2010*

1. Show that there is a way to tile the plane with squares and color them 1 through 7, so that no two points that are distance 1 apart are the same color.
2. Using König's lemma, show that the interval $[0, 1]$ is compact[1]
3. Define a domino as a square of unit area, with one integer attached to every edge, as depicted here:



Define a tiling of some region R in space by some set of dominoes S dominoes as a way of filling up R by dominoes in S, so that adjacent dominoes have the same integer at any edge where they touch.



Using König's lemma, show that the following conditions for a set S of dominoes are equivalent:

- We can tile $\mathbb{R}^2$ with dominoes in S.
- We can tile the upper-right hand quadrant $(\mathbb{R}^+)^2$ with dominoes in S.
- We can tile any $n \times n$-square with dominoes in S.

4. Find the Euclidean dimension of K_n minus an edge.
5. Find the Euclidean dimension of $K_{n,m}$.
6. Find the Euclidean dimension of the wheel graphs W_n.

[1] A set S is called **compact** if for every cover of S by a collection of open intervals $\{(a_i, b_i)\}_{i\in I}$, there is a finite subcover $(a'_1, b'_1), \dots (a'_n, b'_n)$. A **cover** of a set S is a collection of sets $\{A_i\}_{i\in I}$ so that every element in S is also in one of the A_i's.