Flows in Graphs	Instructor: Padraic Bartlett

Homework 1

Week 2	*Mathcamp 2011*

Attempt the problems that seem interesting! Easier exercises are marked with (−) signs; harder ones are marked by (∗). Open questions are denoted by writing (∗∗), as they are presumably quite hard. Oh! Also, typos build character: if you find any (not that there ever could be such things in my problem sets,) correct them to the most reasonable thing you can think of and proceed from there!

1. (a) Hello!

   (b) How are you?

   (c) (−) Given a graph G and abelian group A, we say that G has an **A-circulation** g iff there is some way to assign elements of A to every edge in G, such that we obey Kirchoff's law at every vertex v in G:

   $$\sum_{w \in N^+(v)} g(v,w) - \sum_{w \in N^-(v)} g(w,v) = 0.$$

   Show that the collection of A-circulations on a given graph form a group.

   (d) (−) We say that an A-circulation is in fact an **A-flow** iff $g(e) \neq 0$, for every edge $e \in E(G)$. When does a graph have a $\mathbb{Z}_1$-flow? A $\mathbb{Z}_2$-flow? Do the collection of A-flows also form a group?

   (e) For what values of k does the Petersen graph have a $\mathbb{Z}_k$-flow?

2. (∗) Find a network G, s, t with capacity function c such that Ford-Fulkerson never halts. More dramatically, find such a network where the infintely many flows given by Ford-Fulkerson don't even *converge* to the maximal flow.

3. For any n, find a graph and a rational capacity function such that Ford-Fulkerson takes more than n iterations to find a maximal flow, where you can assume that we choose our augmenting paths however we want.

4. (?) In class, we proved the Max-Flow Min-Cut theorem. Can you formulate and prove a Min-Flow Max-Cut theorem? What would we even mean by this? (We'll discuss this a bit in tomorrow's lecture.)