

Homework 4

Week 1

Mathcamp 2011

1. (*) Suppose that G is the graph with vertex set \mathbb{R}^2 , formed by connecting two points iff they are distance one from each other in the plane. Show that $4 \leq \chi(G) \leq 7$.
2. (**) Determine $\chi(G)$, for G the **unit distance plane graph** defined in the above example.
3. Let G be a k -chromatic graph with girth ≥ 6 , with vertex set $\{v_1, \dots, v_n\}$. Construct a new graph G' as follows:
 - Let T be a set of kn vertices, $\{t_1, \dots, t_{kn}\}$ with no edges between them.
 - Take $\binom{kn}{n}$ disjoint copies of G , one for every n -subset of $\{1, \dots, kn\}$ and index them by these subsets: i.e. for any subset $\{i_1, \dots, i_n\} \subseteq \{1, \dots, kn\}$, make a subgraph $G_{\{i_1, \dots, i_n\}}$.
 - Take each $G_{\{i_1, \dots, i_n\}}$, and connect the vertices of G to the corresponding vertices in T given by G 's indexing subset. In other words, throw in the edges $\{v_1, t_{i_1}\}, \{v_2, t_{i_2}\}, \dots, \{v_n, t_{i_n}\}$ to our graph made by the the G 's and the set T .

Show that this graph still has girth 6, as well as chromatic number ≥ 6 .

4. (–) Using the process above, start with a P_2 and draw the next few graphs created by the above process.
5. Find a construction that shows $R(3, t + 1) > 3t - 1$.
6. Use the picture below to prove that $R(3, 5)$ is 14:

