Martin's Axiom	Instructors: Susan Durst and Padraic Bartlett

Homework 1

Week 5	*Mathcamp 2011*

Attempt the problems that seem interesting!

1. Show that $|\mathbb{R}| = |{}^{\omega}\omega|$, by constructing explicit injections in both directions.

2. Find subsets of ${}^{\omega}\omega$ with cardinality $|\mathbb{R}|$ that **do** have dominating functions.

3. Our definition of dominating functions can be easily extended to the collection of all functions from $\mathbb{R}$ to itself. Can you say anything about which sizes of sets can always be dominated here? Can countable sets of functions always be dominated? Are there conditions you can add on the kinds of functions you consider (continuity et. al.) that might change your answer?

4. Disprove the following claim:

   Claim 1 *If $\mathbb{P}$ is a poset and $\{D_\alpha \mid \alpha < \kappa < |2^\omega|\}$ is a collection of $< |2^\omega|$ dense sets, then there exists a filter $G \subseteq \mathbb{P}$ such that $G \cap D_\alpha \neq \emptyset$ for all $\alpha < \kappa$.*

   This is fairly hard! We offer some hints here:

   - Hint 1: Aim for a contradiction. We just used these posets and filters in order to build a function. Perhaps using a simlar strategy, you can use this axiom to find a function which you know cannot possibly exist.
   - Hint 2: A good contradiction to aim for would be $|\omega| \geq |\omega_1|$.
   - Hint 3: In the previous example, the range of our function was ω. What if we made the range ω_1?
   - Hint 4: Find a poset and a set of dense subsets that will allow us to build an onto function from ω to ω_1.