Perfect Graph Theory | Instructor: Padraic Bartlett

Homework 3

Week 1 | *Mathcamp 2011*

The problems below are completely optional; attempt the ones that seem interesting to you! Easier exercises are marked with $(-)$ signs; harder ones are marked by $(*)$. Open questions are denoted by writing $(**)$, as they are presumably quite hard.

1. $(-)$ Given a collection $I\{I_1, \ldots I_n\}$ of intervals on the real line, define the **interval graph** G_I on the vertex set $\{v_1, \ldots v_n\}$ by drawing an edge $\{v_i, v_j\}$ if and only if $I_i \cap I_j \neq \emptyset$. Prove that any interval graph is perfect.

2. Show that the complement of any interval graph is perfect.

3. Given a permutation $\pi : \{1, \ldots n\} \to \{1, \ldots n\}$, form the **permutation graph** G_π corresponding to π on vertices $\{v_1, \ldots v_n\}$ by connecting v_i to v_j iff π switches the order of i and j. Show that G_π and its complement are perfect.

4. Suppose that we have a graph G with induced subgraphs G_1, G_2, S such that $G = G_1 \cup G_2$ and $S = G_1 \cap G_2$. Then we say that G is the graph given by **pasting** G_1 and G_2 along S.

   Prove that we can create any chordal graph by starting with the set of complete graphs and repeatedly pasting things together.

5. Use the above to prove that chordal graphs are perfect.