

## Homework 3: Partial Latin Squares, continued

Week 2

Mathcamp 2012

Attempt all of the problems that seem interesting, and let me know if you see any typos! (+) problems are harder than the others. (++) problems are currently open.

1. Construct a set of four distinct  $5 \times 5$  MOLS .
2. Construct a set of three distinct  $9 \times 9$  MOLS, as well as a set of three distinct  $8 \times 8$  MOLS.
3. Given a latin square of order  $n$ , must it have an orthogonal mate? (For  $n = 2, 6$ , this is trivially true because there are no pairs of MOLS of order 2 or 6. For other values of  $n$ , can we always make an orthogonal mate? Or for any  $n$ , can you find a Latin square with no orthogonal mate?)
4. (+) Show that there is no pair of  $6 \times 6$  MOLS. (The fastest way to do this is probably to use Mathematica or your favorite programming language to just check cases.)
5. Even though we cannot construct a pair of MOLS of order 6, it turns out that we **can** come pretty close, in the following sense: create a pair of  $6 \times 6$  Latin squares such that when you superimpose these two squares on top of each other, you get 34 distinct pairs of symbols (out of a possible 36 distinct pairs.)
6. (++) Find the size of the largest set of  $10 \times 10$  MOLS.
7. Given a pair of MOLS of order  $m$  and another pair of MOLS of order  $n$ , create a pair of MOLS of order  $mn$ .