

Homework 4: Examples of Quasirandom Graphs

Week 4

Mathcamp 2012

Attempt all of the problems that seem interesting, and let me know if you see any typos!
(-) problems are maybe tedious to attempt. (+) problems are harder than the others.
(++) problems are currently open.

1. Construct the Paley graphs of orders 5,7,11. How many triangles do you see in each of these graphs? Do they look like they're occurring with the "same" frequency as random graphs?
2. Show that the complement of a Paley graph is the same Paley graph.
3. (+) Construct the Paley graph of order 17, and show that neither this graph nor its complement contains a K_4 . (This tells you that the Ramsey number $R(4,4) \geq 18$, by the way. In fact, it turns out that $R(4,4)$ is 18, pretty much because of this graph! Attempt to prove this if you're intrigued.)
4. (+) Show (via computer search via Mathematica, I think) that the Paley graph on 101 elements does not contain a K_6 , and neither does its complement. (This is the best bound we know on $R(6,6)$.)