Math/CCS 103 Professor: Padraic Bartlett

Homework 5: NP-Completeness

Due Friday, week 3 *UCSB 2014*

Homework Problems.

Pick **two** of the problems below, and solve them!

1. Recall that the **degree** of a vertex v in a graph is the number of edges leaving that vertex.

   (a) Prove that if a graph G has each of its vertices with odd degree, then G does not admit a triangulation.

   (b) Prove that if the number of edges in G is not divisible by 3, then G does not admit a triangulation.

   (c) Find a graph G where every vertex has even degree and the number of edges is a multiple of 3, but G does not admit a triangulation.

2. Find a complete graph K_n such that

   - K_n is decomposable into triangles, and
   - $n > 3$.

3. Consider the following game, which you may recognize as Minesweeper:

   Problem. Instance: a $n \times n$ grid, in which each cell has the following properties:

   - Each cell is either **revealed** or **unrevealed**.
   - If a cell is **revealed**, it contains a number.

   Given such a board, we want to find a way of placing **mines** onto some of the unrevealed cells, so that the number on each revealed cell corresponds to the number of its neighbors that contain mines. For example, if a revealed cell was labeled 8, all 8 of its neighboring cells would contain mines.

   We say that a board is **minesweeper-consistent** if there is such a way of placing these mines.

   Create an algorithm for determining whether a given board is minesweeper-consistent. Show that Minesweeper is in NP.