Math 104A - Homework 1

Section 1.1 - 2, 4a, 4c, 8, 12a, 12b, 24

2 Find intervals containing solutions to the following equations.

(a) $x - 3^{-x} = 0$

(b) $4x^2 - e^x = 0$

(c) $x^3 - 2x^2 - 4x + 2 = 0$

(d) $x^3 + 4.001x^2 + 4.002x + 1.101 = 0$

4a Find $\max_{a \le x \le b} |f(x)|$ for $f(x) = (2 - e^x + 2x)/3, x \in [0, 1]$.

4c Find $\max_{a \le x \le b} |f(x)|$ for $f(x) = 2x\cos(2x) - (x-2)^2, x \in [2, 4]$.

8 Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{0.5}, \sqrt{0.75}, \sqrt{1.25}$, and $\sqrt{1.5}$ using $P_3(x)$, and find the actual (absolute and relative) errors.

12a Find the third Taylor polynomial $P_3(x)$ for $f(x) = 2x\cos(2x) - (x-2)^2$ and $x_0 = 0$, and use it to approximate $f(0.4)$.

12b Use the error formula in Taylor's theorem to find an upper bound for the absolute error $|f(0.4) - P_3(0.4)|$. Compute the actual absolute error.

24 The *error function* defined by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

gives the probability that any one of a series of trials will lie within x units of the mean, assuming that the trials have a normal distribution with mean 0 and standard deviation $\frac{\sqrt{2}}{2}$. This integral cannot be evaluated in terms of elementary functions, so an approximating technique must be used.

(a) Integrate the Maclaurin series for e^{-x^2} to show that

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)k!}.$$

(b) The error function can also be expressed in the form

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{k=0}^{\infty} \frac{2^k x^{2k+1}}{1 \cdot 3 \cdot 5 \cdots (2k+1)}.$$

Verify that the two series agree for $k = 1, 2, 3$, and 4. (Hint: Use the Maclaurin series for e^{-x^2}).

(c) Use the series in part (a) to approximate $\operatorname{erf}(1)$ to within 10^{-7}.

(d) Use the same number of terms as in part (c) to approximate $\operatorname{erf}(1)$ with the series in part (b).

(e) Explain why difficulties occur using the series in part (b) to approximate $\operatorname{erf}(1)$.