Math 104A - Homework 2

Due 6/30

Section 1.3 - 1, 7a, 7d, 11
Section 2.1 - 6, 8, 20
Section 2.2 - 5, 8, 14
Section 2.3 - 5, 33

1.3.1a Use three-digit chopping arithmetic to compute the sum $\sum_{i=1}^{10} \frac{1}{i^2}$ first by $\frac{1}{1} + \frac{1}{4} + \cdots + \frac{1}{100}$, and then by $\frac{1}{100} + \frac{1}{81} + \cdots + \frac{1}{1}$. Which method is more accurate, and why?

1.3.1b Write an algorithm (pseudocode) to sum the finite series $\sum_{i=1}^{N} x_i$ in reverse order. (Here, the input is $N, x_1, \ldots, x_N$, and the output is the sum).

1.3.7a Find the rate of convergence of $\lim_{h \to 0} \frac{\sin h}{h} = 1$ (Hint: use Taylor series).

1.3.7b Find the rate of convergence of $\lim_{h \to 0} \frac{1-e^h}{h} = -1$.

1.3.11 Construct an algorithm (pseudocode) that has as input an integer $n \geq 1$, numbers $x_0, x_1, \ldots, x_n$, and a number x that produces as output the product $(x - x_0)(x - x_1) \cdots (x - x_n)$.

2.1.6 Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems:

- **a** $3x - e^x = 0, x \in [1, 2]$.
- **b** $x + 3\cos x - e^x = 0, x \in [0, 1]$.
- **c** $x^2 - 4x + 4 - \ln x = 0, x \in [1, 2]$ and $x \in [2, 4]$.
- **d** $x + 1 - 2\sin(\pi x) = 0, x \in [0, 0.5]$ and $x \in [0.5, 1]$.

2.1.8a Sketch the graphs of $y = x$ and $y = \tan x$.

2.1.8b Use the bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = \tan x$.

2.1.20 A particle starts at rest on a smooth inclined plane whose angle θ is changing at a constant rate $\frac{d\theta}{dt} = \omega < 0$. At the end of t seconds, the position of the object is given by

$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin(\omega t)\right).$$

Suppose the particle has moved 1.7 ft in 1 s. Find, to within 10^{-5}, the rate ω at which θ changes. Assume that $g = 32.17$ ft/s^2.

2.2.5 Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

2.2.8 Use theorem 2.2 to show that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$. Use fixed-point iteration to find an approximation to the fixed point accurate to within 10^{-4}. Use corollary 2.4 to estimate the number of iterations required to achieve 10^{-4} accuracy, and compare this theoretical estimate to the number actually needed.

2.2.14 Use a fixed-point iteration method to determine a solution accurate to within 10^{-4} for $x = \tan x$, for $x \in [4, 5]$.

2.3.5 Use Newton's method to find solutions accurate to within 10^{-4} for the following problems:

a $x^3 - 2x^2 - 5 = 0, x \in [1, 4]$.

b $x^3 + 3x^2 - 1 = 0, x \in [-3, -2]$.

c $x - \cos x = 0, x \in [0, \pi/2]$.

d $x - 0.8 - 0.2 \sin x = 0, x \in [0, \pi/2]$.

2.3.33 Player A will shut out (win by a score of 21-0) player B in a game of raquetball with probability

$$P = \frac{1+p}{2} \left(\frac{p}{1 - p + p^2} \right)^{21},$$

where p is the probability that A will win any specific rally (independent of the server). Determine, to within 10^{-3}, the minimal value of p that will ensure that A will shut out B in at least half the matches they play.