Math 104A - Homework 3

Due 7/7

Section 2.4 - 6, 7a, 8, 9, 11
Section 3.1 - 6, 11, 19ab, 26
Section 3.2 - 2, 19
Section 3.3 - 2, 4, 10
Section 3.4 - 30

2.4.6 Show that the following sequences converge linearly to $p = 0$. How large must n be before we have $|p_n - p| \leq 5 \cdot 10^{-2}$?

- **a** $p_n = 1/n$.
- **b** $p_n = 1/n^2$.

2.4.7a Show that for any positive integer k, the sequence defined by $p_n = 1/n^k$ converges linearly to $p = 0$.

2.4.8a Show that the sequence $p_n = 10^{-2^n}$ converges quadratically to 0.

2.4.8b Show that the sequence $p_n = 10^{-n^k}$ does not converge quadratically, regardless of the size of the exponent k.

2.4.9a Construct a sequence that converges to 0 of order 3.

2.4.9b Suppose $\alpha > 1$. Construct a sequence that converges to 0 of order α.

2.4.11 Show that the bisection method gives a sequence with an error bound that converges linearly to 0.

3.1.6 Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following: Note: You can do these by hand, but I highly suggest implementing Neville's iterated interpolation.

- **a** $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$
- **b** $f(0)$ if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, $f(0.5) = 0.687500$
- **c** $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$
- **d** $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$, $f(0) = 1.0986123$, $f(0.5) = 1.2943767$

3.1.11 Use Neville's method to approximate $\sqrt{3}$ with the following functions and values.

- **a** $f(x) = 3^x$ and the nodes $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$.
- **b** $f(x) = \sqrt{x}$ and the nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, $x_4 = 5$.

c Compare the accuracy of the approximation in parts (a) and (b).

3.1.19 Construct the Lagrange interpolating polynomails for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

a $f(x) = e^{2x}\cos(3x)$, $x_0 = 0, x_1 = 0.3, x_2 = 0.6$.

b $f(x) = \sin(\ln x)$, $x_0 = 2, x_1 = 2.4, x_2 = 2.6$

3.1.26 **Inverse Interpolation** Suppose $f \in C^1[a,b], f'(x) \neq 0$. Let $x_0, \ldots, x_n$ be $n+1$ distinct numbers in $[a,b]$ with $f(x_k) = y_k$. To approximate the root p of f, construct the interpolating polynomial of degreen n on the nodes $y_0, \ldots, y_n$ for the function f^{-1}. Since $y_k = f(x_k)$ and $0 = f(p)$, it follows that $f^{-1}(y_k) = x_k$ and $f^{-1}(0) = p$. Using iterated interpolation to approximate $f^{-1}(0)$ is called *iterated inverse interpolation*.

Use iterated inverse interpolation to find an approximation to the solution of $f(x) = x - e^{-x} = 0$, using the data

x	0.3	0.4	0.5	0.6
e^{-x}	0.740181	0.670320	0.606531	0.548812

3.2.2 Use Algorithm 3.2 (Newton's divided differences) to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

b $f(0)$ if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, $f(0.5) = 0.687500$

3.2.19 Given

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1) \\ + \cdots + a_n(x - x_0)\cdots(x - x_{n-1}),$$

use $P_n(x_2)$ to show that $a_2 = f[x_0, x_1, x_2]$.

3.3.2 Use algorithm 3.3 (Hermite interpolation) to construct an approximating polynomial for the following data.

a

x	$f(x)$	$f'(x)$
0	1	2
0.5	2.71828	5.43656

b

x	$f(x)$	$f'(x)$
-0.25	1.33203	-1.7500
0.25	0.800781	-0.625000

c

x	$f(x)$	$f'(x)$
0.1	-0.29004996	-2.8019975
0.2	-0.56079734	-2.6159201
0.3	-0.81401972	-2.4533949

	x	$f(x)$	$f'(x)$
	-1	0.86199480	0.15536240
d	-0.5	0.95802009	0.2326954
	0	1.0986123	0.33333333
	0.5	1.2943767	0.45186776

3.3.4 The data in 3.3.2 were generated using the following functions. Use the polynomials constructed in 3.3.2 for the given value of x to approximate $f(x)$, and calculate the absolute error.

a $f(x) = e^{2x}$; approximate $f(0.43)$.

b $f(x) = x^4 - x^3 + x^2 - x + 1$; approximate $f(0)$.

c $f(x) = x^2 \cos(x) - 3x$; approximate $f(0.18)$.

d $f(x) = \ln(e^x + 2)$; approximate $f(0.25)$.

3.3.10 A car traveling along a straight road is clocked at a number of points. The data from the obsercations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

a Use a Hermite polynomial to predict the position of the car and it's speed when $t = 10$ s.

b Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?

c What is the predicted maximum speed for the car?

3.4.30 The 2004 Kentucky Derby was won by a horse named Smarty Jones in a time of 2:04.06 (2 minutes and 4.06 seconds) for the $1\frac{1}{4}$-mile race. Times at the quarter-mile, half-mile, and mile poles were 0:22.99, 0:46.73, and 1:37.35.

a Use these values together with the starting time to construct a free cubic spline for Smarty Jones' race.

b Use the spline to predict the time at the three-quarter-mile pole, and compare this to the actual time of 1:11.80.

c Use the spline to approximate Smarty Jones' starting speed and speed at the finish line.