Math 104A - Homework 6 (Extra Credit)

Due 7/28

1 Write a code that implements the order four Runge-Kutta method (algorithm 5.2 in the book). Below is an example code template:

```
function [t,w] = RK4(a,b,alpha,N,fstring)

    f = inline(fstring,'t','y');
    t = linspace(a,b,N+1);
    h = (b-a)/N;

    w = zeros(size(t));
    w(1) = alpha;

    %%%%% Code for order 4 Runge-Kutta goes here %%%%%

end
```

2 Test your code on the initial value problem

$$y' = -y + \sin(t), 0 \leq t \leq 10, y(0) = 1,$$

which has exact solution $y(t) = \frac{1}{2}(3e^{-t} + \sin(t) - \cos(t))$. Using a stepsize $h = \frac{1}{50}$ (i.e. $N = 500$), give a plot of your approximate solution and the exact solution. **Be sure to label your plots clearly**.

3 Suppose you've found that a good model for the outside temperature during an average day is given by

$$\tau(t) = 19[\tanh(5\sin(2\pi t)) + 1] + 60,$$

where τ is in °F and t is measured in days. According to Newton's law of cooling, the change in the temperature T in your house is proportional to the difference in the outside temperature τ and the inside temperature. That is,

$$T'(t) = -k(T(t) - \tau(t)),$$

where k depends on what kind of insulation you use in your house. Use your code to plot the temperature in your house over three days, assuming that the temperature at the start of the first day is 79 °F. Use a step size $h = 1/100$, and give a plot for $k = 2, 5$, and 10. Which kind of insulation would you use (corresponding to which value of k)? Why?