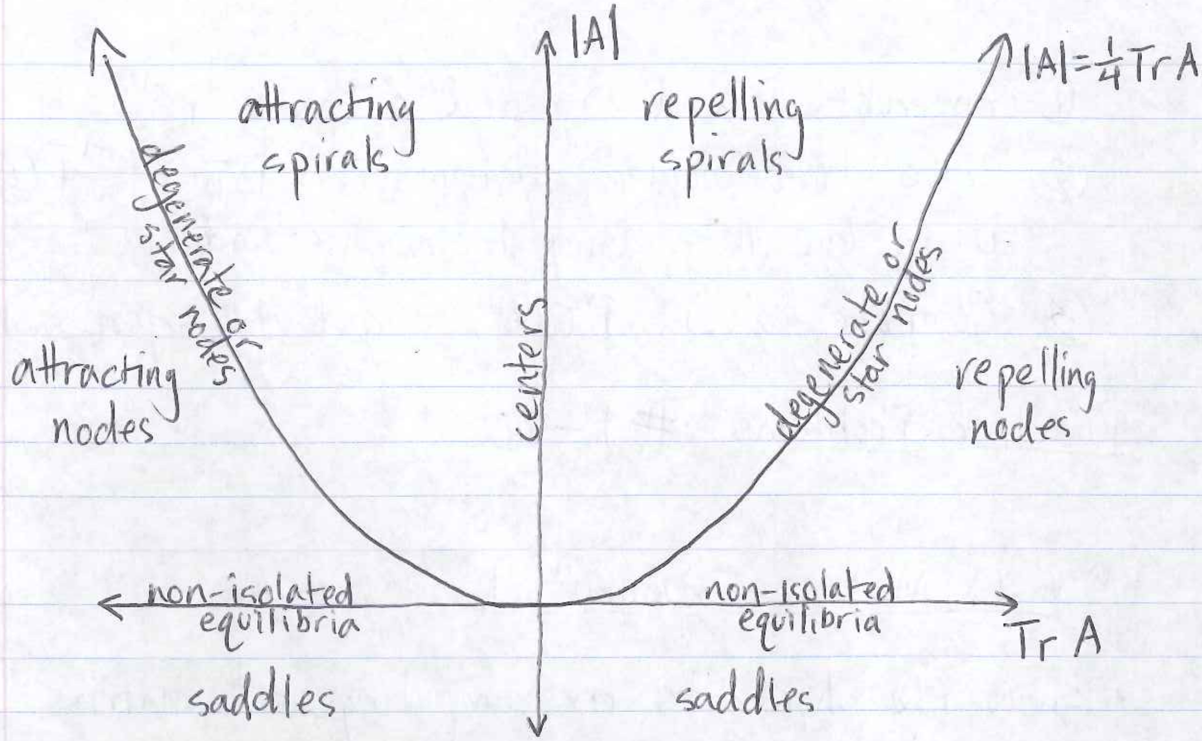

<u>Practice Problems</u>: #1-6

6.5 Decoupling a Linear DE System

- To decouple a system $\vec{x}' = A\vec{x}$:
  1. Make sure A is diagonalizable (i.e. has a full set of linearly independent e-vectors)
  2. Construct the matrices P and D
  3. Solve the decoupled system $\vec{u}' = D\vec{u}$
  4. Compute $\vec{x}(t) = P\vec{u}(t)$ to get the general solution

  - Check out the examples in the notes online (7/26)

- Decoupling a non-homogeneous system $\vec{x}' = A\vec{x} + \vec{f}(t)$:

① Construct P and D, and compute P^{-1}

② Solve the decoupled system $\vec{u}' = D\vec{u} + P^{-1}\vec{f}(t)$ using the integrating factor method

③ Compute $\vec{x}(t) = P\vec{u}(t)$ to get the gen. soln.

Practice Problems: #1-20

6.6 Matrix Exponential

- Know the shortcuts for computing the matrix exponential:
  - if A is diagonal:

$$A = \begin{bmatrix} a_{11} & & 0 \\ & a_{22} & \\ 0 & \ddots & a_{nn} \end{bmatrix} \Rightarrow e^A = \begin{bmatrix} e^{a_{11}} & & 0 \\ & e^{a_{22}} & \\ 0 & \ddots & e^{a_{nn}} \end{bmatrix}$$

  - if A is nilpotent, i.e. $A^n = 0$ for some n:

$$e^A = I + A + \frac{A^2}{2!} + \cdots + \frac{A^n}{n!}$$

- Know the 3 ways to compute e^{At}:

① The series definition:

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots$$

(usually the most difficult way)

② Find a fundamental matrix for the equation $\vec{x}' = A\vec{x}$. Then

$$e^{At} = X(t)X(0)^{-1}$$

(3) If A is diagonalizable, then:

$$e^{At} = Pe^{Dt}P^{-1}$$

- know that you can use matrix exponentials to solve non-homogeneous equations: given

$$\vec{x}' = A\vec{x} + \vec{f}(t), \quad \vec{x}(0) = \vec{x}_0$$

the general solution is

$$\vec{x}(t) = e^{At}\vec{x}_0 + e^{At}\int_0^t e^{-As}\vec{f}(s)\,ds$$

Practice Problems: #1-14

6.7 Nonhomogeneous Linear Systems

- know how to extend the method of undetermined coefficients to systems:

Ex: if $\vec{f}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \vec{x}_p = \begin{bmatrix} a \\ b \end{bmatrix}$

if $\vec{f}(t) = \begin{bmatrix} t-2 \\ 4t+1 \end{bmatrix} \rightarrow \vec{x}_p = \begin{bmatrix} at+b \\ ct+d \end{bmatrix}$

if $\vec{f}(t) = \begin{bmatrix} \cos t \\ -2\sin t \end{bmatrix} \rightarrow \vec{x}_p = \begin{bmatrix} a\cos t + b\sin t \\ c\cos t + d\sin t \end{bmatrix}$

if $\vec{f}(t) = \begin{bmatrix} e^{3t} \\ 0 \end{bmatrix} \rightarrow \vec{x}_p = \begin{bmatrix} ae^{3t} \\ be^{3t} \end{bmatrix}$

- solve the homogeneous system first: $\vec{x}' = A\vec{x}$, and then substitute the guess for $\vec{x}_p$ to determine the coefficients

- know how to use variation of parameters for a system of DE's: given $\vec{x}' = A\vec{x} + \vec{f}(t)$, $\vec{x}(0) = \vec{x}_0$:

① Find a fundamental matrix $X(t)$ for the problem $\vec{x}' = A\vec{x}$

② Compute $X(t)^{-1}$, and then compute

$$\int_0^t X(s)^{-1}\vec{f}(s)\,ds$$

③ The exact solution is

$$\vec{x}(t) = X(t)\,X(0)^{-1}\vec{x}_0 + X(t)\int_0^t X^{-1}(s)\,\vec{f}(s)\,ds$$

<u>Practice Problems:</u> #4 - 16