The Grand Table of Integration Concepts for Math 5B

Type of Integration Domain

Concept	Intervals	Curves	Regions	Surfaces
Integrals of real-valued functions	$\int_I f(t)\ dt$	$\int_{\mathbf{c}} f\ ds = \int_I f(\mathbf{c}(t))\|\mathbf{c}'(t)\|\ dt$	$\iint_D f(x,y)\ dA$	$\iint_S f\ dS = \iint_D f(\mathbf{r}(u,v))\|\mathbf{N}(u,v)\|\ dA$
Integrals of vector-valued functions		$\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_I \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\ dt$		$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot\mathbf{N}(u,v)\ dA$
Length/Area	Length of interval $= \int_I\ dt$	Arclength $= \int_{\mathbf{c}}\ dt = \int_I \|\mathbf{c}'(t)\|\ dt$	Area of a region $= \iint_D\ dA$	Surface Area $= \iint_S\ dS$ $= \iint_D \|\mathbf{N}(u,v)\|\ dA$
Fundamental Theorem of Calculus	$\int_I \frac{d}{dt}f(t)\ dt = f(b) - f(a)$	$\int_{\mathbf{c}} \nabla f\cdot ds = f(\mathbf{c}(b)) - f(\mathbf{c}(a))$	$\iint_D \nabla\times\mathbf{F}\ dA = \int_{\partial D}\mathbf{F}\cdot d\mathbf{s}$ $\iint_D \nabla\cdot\mathbf{F}\ dA = \int_{\partial D}\mathbf{F}\cdot\mathbf{n}\ ds$	$\iint_S \nabla\times\mathbf{F}\cdot d\mathbf{S} = \int_{\partial S}\mathbf{F}\cdot d\mathbf{s}$
Physics		$\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \text{work}$		$\iint_S \mathbf{F}\cdot d\mathbf{S} = \text{flux}$
Change of Variables	$\int_I f(x)\ dx = \int_{I^*} f(x(u))\left\|\frac{\partial x}{\partial u}\right\|\ du$		$\iint_D f(x,y)\ dA$ $= \iint_{D^*} f(T(u,v))\left\|\frac{\partial(x,y)}{\partial(u,v)}\right\|\ dA^*$	
Dependence on Parametrization (Real-valued Functions)		$\int_{I_1} f(\mathbf{c}_1(t))\|\mathbf{c}_1'(t)\|\ dt = \int_{I_2} f(\mathbf{c}_2(t))\|\mathbf{c}_2'(t)\|\ dt$		$\iint_{D_1} f(\mathbf{r}_1(u,v))\|\mathbf{N}_1(u,v)\|\ dA$ $= \iint_{D_2} f(\mathbf{r}_2(u,v))\|\mathbf{N}_2(u,v)\|\ dA$
Dependence on Parametrization (Vector-valued Functions)		$\int_{I_1} \mathbf{F}(\mathbf{c}_1(t))\cdot\mathbf{c}_1'(t)\ dt = \pm\int_{I_2} \mathbf{F}(\mathbf{c}_2(t))\cdot\mathbf{c}_2'(t)\ dt$		$\iint_{D_1} \mathbf{F}(\mathbf{r}_1(u,v))\cdot\mathbf{N}_1(u,v)\ dA$ $= \pm\iint_{D_2} \mathbf{F}(\mathbf{r}_2(u,v))\cdot\mathbf{N}_2(u,v)\ dA$