Some Graphing Help

There are three basic tricks (that I know of) for figuring out what a given equation corresponds to:

1. Figure out what its level curves look like, i.e., set z = C and plot the corresponding curve.

2. Graph its intersection with the x - z plane or the y - z plane by setting y or x equal to 0, respectively. You can also set x or y equal to any other constant for even more information.

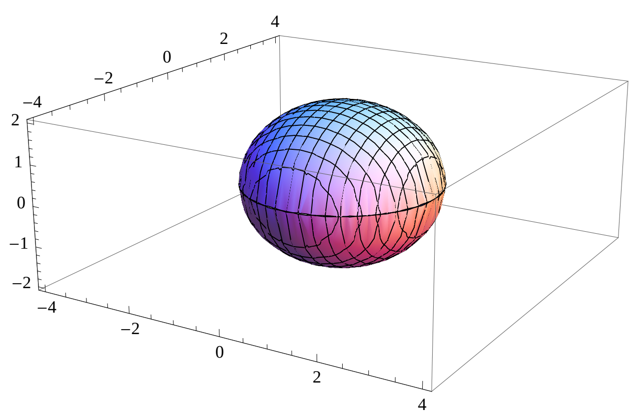
3. Use Mathematica or Wolfram Alpha (http://www.wolframalpha.com/)

Of course, these tricks don't cover everything. There are a lot of strange functions out there. Sometimes you just gotta be creative.

Here are some basic surfaces you should become familiar with:

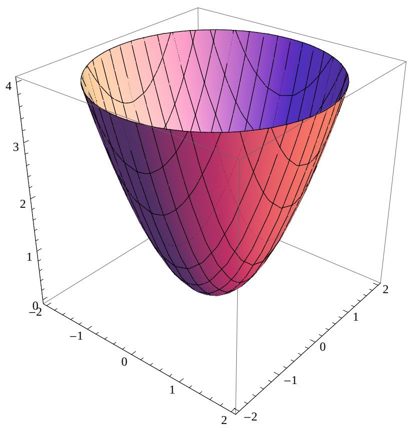
1. Planes

2. Sphere of radius r: $r^2 = x^2 + y^2 + z^2$.



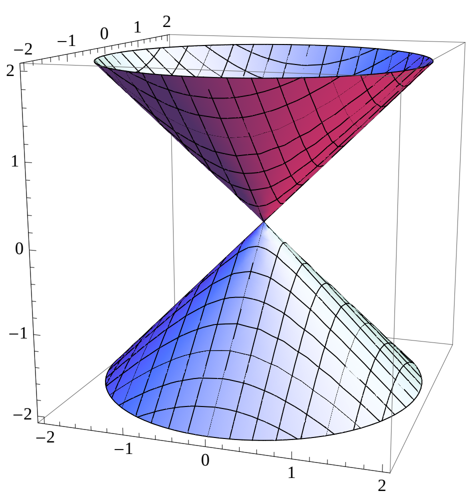
3. Elliptic Parabaloid: $z = ax^2 + by^2$, where $a, b > 0$.

   - Level curves are circles or ellipses
   - Intersection with x - z or y - z plane is a parabola.

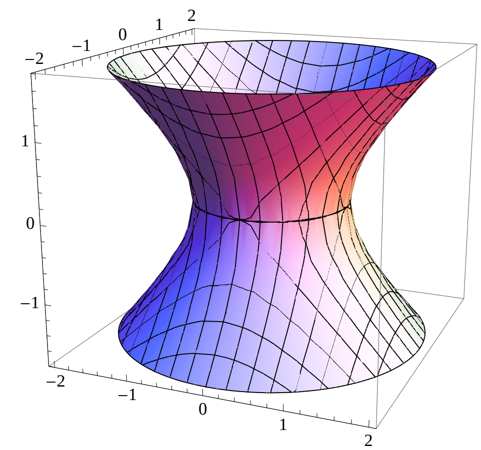


4. A pair of Cones: $z^2 = ax^2 + by^2$, where $a, b > 0$

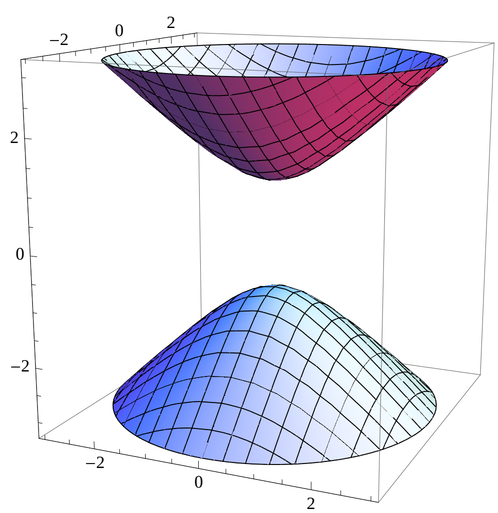
   - Level curves are circles or ellipses
   - Intersection with x - z or y - z plane is a pair of lines



5. Hyperboloid of One Sheet: $ax^2 + by^2 - cz^2 = 1$, where $a, b, c > 0$
   - Level curves are circles or ellipses
   - Intersection with x - z or y - z plane is a hyperbola, opening in the x or y direction respectively.



6. Hyperboloid of Two Sheets: $-ax^2 - by^2 + cz^2 = 1$, where $a, b, c > 0$
   - Notice that if $c = 1$ and $-1 < z < 1$, then this equation has no solution.
   - Level curves are circles or ellipses
   - Intersection with x - z or y - z plane is a hyperbola opening in the z direction.



Recall that in old-fashioned two-dimensional graphing, if we swap x and y our curve changes by a reflection about the line $y = x$. Likewise, if we swap any two of the variables x, y, and z, our surface changes by a reflection about an appropriate plane. For example, if you want a paraboloid that opens in the x-direction, just swap z and x in the equation above.

A Word About Directional Derivatives

Recall the definition of a partial derivative with respect to x at the point (a, b):

$$\frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a + h, b) - f(a, b)}{h}$$

We can interpret this as the rate of change in $f(x, y)$ at (a, b) **along the line** $y = 0$. If we want to find the rate of change along a different line, say $y = mx$, then we can take a very similar limit:

$$\frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a + h, b + mh) - f(a, b)}{h}$$

The directional derivative of $f(x, y)$ at (a, b) in the direction of a vector $\mathbf{u} = (u_1, u_2)$ can be thought of as the rate of change of f along the line in the direction of $\mathbf{u}$. This leads to the definition of the directional derivative:

$$\frac{\partial f}{\partial x}(a, b) = \lim_{h \to 0} \frac{f(a + u_1 h, b + u_2 h) - f(a, b)}{h}$$