Hand-in Homework

Work out the following problems on your own paper and hand them in during class on Friday. Show all your work!

1. Consider the vector field $F = (3 + 2xy + z^2, x^2 + 2y, 2xz)$

   (a) Find $Curl F$

   (b) Can you conclude that F is a gradient vector field? Why or why not?

   (c) Find the value of $\int_\gamma F \cdot ds$ where γ is the helicoid $\gamma(t) = \left(\cos t,\ \sin t,\ \frac{1}{10\pi}t\right)$ from $t = 0$ to $t = 10\pi$.

   Hint: If you're evaluating a complicated integral, you're doing something wrong.

2. Determine whether the domains of the following functions are simply connected. Why or why not?

   (a) $f(x, y) = \frac{1}{x^2 + y^2}$

   (b) $f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$

   (c) $F(x, y, z) = \left(x^2,\ 3y,\ \frac{1}{x^2 + y^2}\right)$

3. Let $F(x, y) = (2e^{2x} + 4xy, 2x^2 + 3y^2)$ be a vector field. It is gradient (you can verify this by checking $Curl F$ if you'd like). In this problem, you'll learn a technique for finding $f : \mathbb{R}^2 \to \mathbb{R}$ such that $\bigtriangledown f = F$.

   (a) Since we know $\frac{\partial}{\partial x} f = 2e^{2x} + 4xy$, we can take the antiderivative with respect to x to get an expression for $f(x, y)$ up to a function involving only y. Think: In single variable calc, we always had to include a "+ C". If we integrate a function of two variables x and y with respect to x, then we have to include a "+ $g(y)$" since y is taken to be a constant.

   Your answer to this part should be of the form $f(x, y) = \text{stuff} + g(y)$, where $g(y)$ represents an unknown function of y.

   (b) We now have an expression $f(x, y)$ except for the $g(y)$ term. To solve for $g(y)$, first take the partial derivative with respect to y of the function you found in part (a).

   Your answer should be of the form $\frac{\partial}{\partial y} f(x, y) = \text{stuff} + g'(y)$.

   (c) Now set the answer you got in part (b) equal to $2x^2 + 3y^2$ (why?) and solve for

$g'(y)$.

(d) Find $g(y)$ by integrating $g'(y)$ the old fashioned way. Plugging in to your answer in part (a), you have found an expression for $f(x,y)$!

(e) Verify you got the correct answer by checking that $\bigtriangledown f(x,y) = F(x,y)$.