Hand-in Homework

Work out the following problems on your own paper and hand them in during class on Friday. Show all your work!

1. Set up, but do not evaluate, the double integral you would take to find $\iint_D f\, dA$ for each given $f(x,y)$ and region D.

   (a) $f(x,y) = xy$, and let D be the region between the x-axis and the graph of e^x between $x = 0$ and $x = 3$.

   (b) $f(x,y) = \dfrac{1}{x^2+y^2}$ and let D be the triangle with vertices $(-1,0)$, $(-1,1)$, and $(1,1)$.

   (c) $f(x,y) = e^{x^2+y^2}$ and let $D = \{(x,y) \mid x \geq 0,\ 2x \leq y \leq 4\}$

   (d) $f(x,y) = x+3y$ and let D be the region between the curves $x = y^2-2$ and $y = x$.

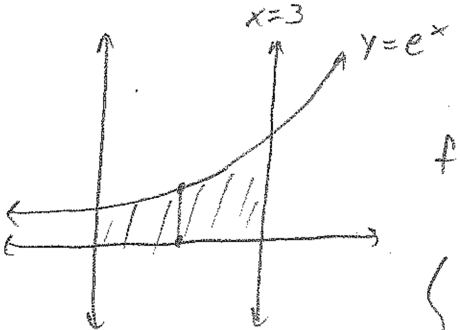
2. Set up, but do not evaluate, the triple integral would would take to find $\iiint_W f\, dV$ where $f(x,y,z) = x^2 - y + z$ and W is the solid bounded by the surface $z = x^2$ and the planes $z = 3$, $y = -1$, and $y = 1$.

3. Set up, but do not evaluate, in two different ways the triple integral you would use to find the volume of the region in the first octant bounded by the plane $2x + 2y + z = 8$. (Hint: if you're having trouble visualizing this, go back to the example we did in class or your book for a helpful picture).

4. Compute the path integral $\int_\gamma F \cdot ds$ where $F(x,y) = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2}\right)$ and γ is the circle of radius 1 oriented counterclockwise. Do this in two different ways, once by evaluating the path integral, and once by using Green's theorem. Why are your answers different? Which answer is the correct one? Why?

5. Use Green's theorem to evaluate $\int_\gamma F \cdot ds$ where $F(x,y) = (3x, -2y)$ and γ is the boundary of the rectangle with vertices $(0,0), (3,0), (0,1)$, and $(3,1)$.

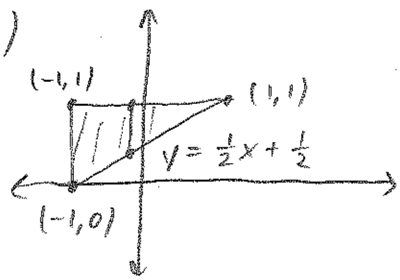
1a)



$$f(x,y) = xy$$

$$\iint_D f\,dA = \int_0^4 \int_0^{e^x} xy\ dy\ dx$$

1b)

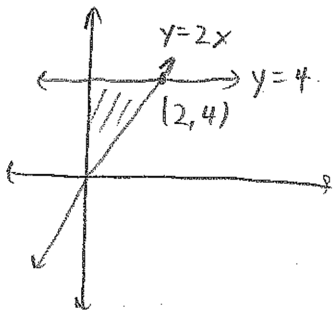


$$f(x,y) = \frac{1}{x^2+y^2}$$

$$\iint_D f\,dA = \int_{-1}^{1} \int_{\frac{1}{2}x+\frac{1}{2}}^{1} \frac{1}{x^2+y^2}\ dy\ dx$$

or: $$\iint_D f\,dA = \int_0^1 \int_{-1}^{2y-1} \frac{1}{x^2+y^2}\ dx\ dy$$

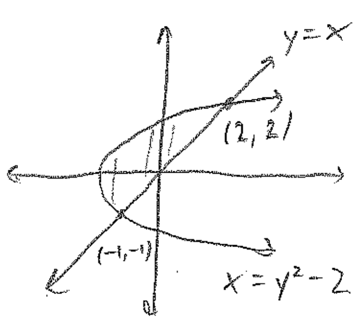
1c)



$$f(x,y) = e^{x^2+y^2}$$

$$\iint_D f\,dA = \int_0^2 \int_{2x}^{4} e^{x^2+y^2}\ dy\ dx$$

1d)

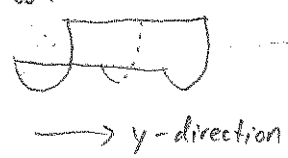


$$f(x,y) = x+3y$$

$$\iint_D f\,dA = \int_{-1}^{2} \int_{y^2-2}^{y} x+3y\ dx\ dy$$

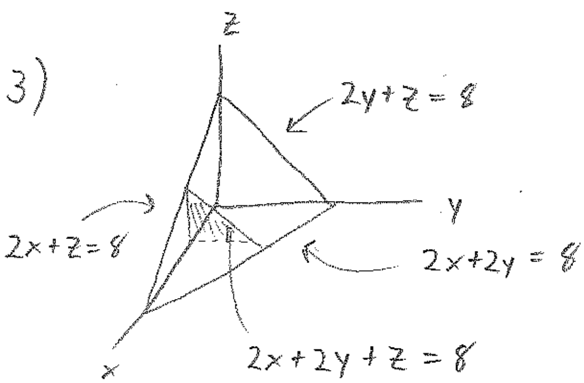
2) looks like a trough

W:



$$\iiint_W f\,dV = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-1}^{1} \int_{x^2}^{3} x^2 - y + z \; dz\,dy\,dx$$

3)

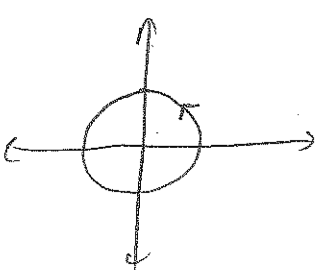


$$\iiint_W f\,dV$$

$$= \int_0^4 \int_0^{4-x} \int_0^{8-2x-2y} 1 \; dz\,dy\,dx$$

$$= \int_0^4 \int_0^{8-2x} \int_0^{4-x-\frac{z}{2}} 1 \; dy\,dz\,dx$$

4) $F(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$, $\gamma(t) = (\cos t, \sin t)$



Green's: $Q(x,y) = \frac{x}{x^2+y^2}$

$$\frac{\partial Q}{\partial x} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$P(x,y) = \frac{-y}{x^2+y^2}$$

$$\frac{\partial P}{\partial y} = \frac{-(x^2+y^2)+2y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$\longrightarrow$

$$\int_\gamma F \cdot ds = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = 0. \quad \text{By Greens.}$$

But:

$$\int_\gamma F \cdot ds = \int_0^{2\pi} (-\sin t, \cos t) \cdot (-\sin t, \cos t)\, dt$$

$$= \int_0^{2\pi} 1\, dt = 2\pi.$$

This is the correct answer. Green's Thm does not apply because $F(x,y)$ is not C^1 on D (in fact, not even cont. on D).

5) $F(x,y) = (3x, -2y)$

$$\frac{\partial Q}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = 0$$

So $\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = 0.$