Hand-in Homework

Work out the following problems on your own paper and hand them in during class on Wednesday. Show all your work!

1. Let S be the surface defined by $z = xy$.
   (a) Find a parameterization $r(u, v)$ for S and find the normal vector field $N = T_u \times T_v$.
   (b) Find a normal vector field for S by taking the gradient of a function of three variables.
2. Let S be a surface, and let $\gamma : [a, b] \to \mathbb{R}^3$ be a curve on S. Say N is a normal vector field for S. What is the value of $\int_\gamma N \cdot ds$? Why?
3. Let S be the cylinder parameterized by $r(u, v) = (\cos u, \sin u, v)$, $0 \leq u \leq 2\pi$, $0 \leq v \leq 3$. For each vector field below, determine without doing any calculations whether the flux integral $\iint_S F \cdot dS$ is zero or nonzero. Why?
   (a) $F(x, y, z) = (x, y, 0)$
   (b) $F(x, y, z) = (1, 0, 0)$
   (c) $F(x, y, z) = (0, 0, xyz)$
4. The surface of revolution S formed by rotating $f(x) = \dfrac{1}{x}$, $x \geq 1$ about the x-axis is called *Gabriel's Horn*.
   (a) Find a parameterization for S.
   (b) Show that the surface area of S is infinite (Hint: you'll set up an improper integral and have to take a limit as something approaches ∞. If you're doing a hard integral, you're doing something wrong. There's a trick.)
   (c) Find the volume of S, using a double integral or math 3B tricks. Weird! This is called the "Painter's Paradox."
5. Find $\iint_S F \cdot dS$ where S is the part of the plane $x + 2y + 8z = 8$ in the first octant and $F(x, y, z) = (x^2 y, -x - y, -z^2 x)$.